THE UNIVERSITY OF ALBERTA

MATHEMATICS LEARNING AND PUPIL CHARACTERISTICS

by

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ABSTRACT

The purpose of this study was to examine the relationship between certain pupil characteristics and mathematics learning in three distinct settings. The characteristics studied included intellectual ability, reading ability, reflectiveness, impulsiveness, socio-economic status, ability to make personal adjustment, ability to make social adjustment and sex.

Fifty-one subjects were assigned to a self-directed setting. They worked in groups of four and received no formal instruction from a teacher. A group of fifty-three subjects was placed in a partially teacher-directed setting. The pupils in this treatment were formally introduced to the mathematical topics and then they worked in groups just as the self-directed subjects. The third group consisted of thirty-seven pupils. Their work was teacher-directed and they completed tasks on an individual basis completely under the control of the teacher.

Mathematics materials for pupils and teachers were prepared in advance and for the duration of the study, which lasted four weeks; these materials formed the basis of the mathematics program.

After the completion of the study an Initial Learning Test was administered to all of the subjects. The same test was again given four weeks later to obtain a measure of retention.

Intercorrelations between the variables and the criteria were calculated and the most important findings are as follows:

(1) For the subjects in the self-directed setting none of the pupil characteristics considered could be used to predict the learning outcomes as measured on the Initial Learning and Retention Test. There

existed no differences in means on these two tests between boys and girls and between subjects who were classified as reflective or impulsive.

(2) For the subjects in the partially teacher-directed setting there existed significant relationships between the criterion variables initial learning and retention, and the following factors: intelligence, personal adjustment, social adjustment and reading ability. Of these factors, only intelligence showed a significant relationship with the pretest scores. The mean for the reflective subjects on the Retention Test was significantly higher than the mean for the impulsive subjects. The means for the boys and girls did not differ.

(3) The learning outcomes for the teacher-directed subjects could be predicted on the basis of intellectual ability only. There existed no differences in means on the two criteria between boys and girls and between subjects who were classified as reflective or impulsive.

The report of the study is concluded with implications for educational practice and suggestions for further research.

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CHAPTER I

THE PROBLEM, ITS NATURE AND SIGNIFICANCE

I. INTRODUCTION

Major changes in the content and approach of elementary school mathematics have occurred during recent years. The content is being modified, restructured and reorganized and as a result elementary school pupils of today are introduced to concepts that were previously presented to older students in higher grades. Less emphasis is placed on mechanically-learned arithmetic and children are presented a range of important concepts of mathematics that can be applied in a wide variety of situations.

It has been frequently stated that new or revised content alone will not guarantee successful learning. The content of contemporary mathematics programs might differ with respect to order or sequence. Nevertheless, these programs have one common factor. A strong emphasis is placed on the method of presentation and the learning process.

Such labels as experience or activity approach, individualized instruction, independent or directed discovery and mathematics laboratories are used to describe a variety of approaches to instruction in mathematics. No matter which label is used, there exists one common characteristic for all of them and that is the active involvement of the learner. Students are provided with various mathematical activities such as games, puzzles, problems, or experiments that help them discover for themselves mathematical principles, relationships or processes through active participation.

The major difference between a 'traditional' program and a setting which includes some active learning situations is the shift from an authoritarian, teacher-dominated classroom to one which is more child-centered. The first program tends to emphasize content; the latter uses experiences of children to build concepts and strategies. An attempt is made to increase pupil involvement. At the same time the classroom teacher has to adapt himself to a new role.

In the content-oriented setting the teacher was the central figure in the classroom and his main concern was to explain examples, assign and check practice items and to cover the material assigned to a specific grade level. In his new role the teacher selects and prepares activities and instead of 'teaching' or lecturing a whole class, he provides the focus of discussion for the individual or a small group.

As a result of these changes there also exists a new role for the elementary school pupil in the mathematics classroom. He is given an opportunity to make more of his own investigations and draw his own inferences. He is guided by the teacher, and given a chance to discover as much as possible on his own. His work is becoming more self-directed.

In this study an attempt was made to measure the effectiveness of a self-directed, a partially teacher-directed and a teacher-directed treatment for different learners. Some characteristics of pupils which might be favorable or unfavorable for the approaches to the teaching of mathematics listed above were examined, and the learning outcomes were explored.

II. STATEMENT OF THE PROBLEM

The major purposes of this study were:

(1) to isolate some pupil characteristics which might facilitate learning of mathematics and to examine the role of these characteristics in a self-directed, a partially teacher-directed, and a teacher-directed setting.

(2) to evaluate the learning outcomes of a self-directed mathematics program.

(3) to evaluate the learning outcomes in a setting that is partially teacher-directed.

(4) to evaluate the learning outcomes in a purely teacher-directed setting.

With respect to these purposes some mathematical topics were selected on the basis of suitability and probable unfamiliarity to the subjects. These topics were presented to three groups of subjects.

Self-Directed (SD): The subject in this group worked in groups of three or four without any direct instruction from the teacher. The teacher's main task consisted of providing the focus of discussion for the groups and answering questions initiated by the pupils.

Partially Teacher-Directed Group (PTD): For this group the teacher provided an introduction for each topic. For the remainder of the time the subjects worked in small groups just as the SD-subjects.

Teacher-Directed Group (TD): The setting for these subjects was somewhat more 'traditional'. The topics were the same as for the SD- and the PTD-groups. The teacher introduced the topics and worked out examples. He assigned tasks which were completed and checked on an individual basis.

A number of characteristics of the students may affect the ability to learn mathematics. For the first purpose of the study the following characteristics were considered: intellectual ability, reading achievement, conceptual tempo (impulsiveness-reflectiveness), socio-economic status, sex, personal adjustment (self-reliance, sense of personal worth, sense of personal freedom, feeling of belonging, freedom from withdrawing tendencies, freedom from nervous symptoms), and social adjustment (social standards, social skills, freedom from anti-social tendencies, family relations, school relations, community relations).

Basically, this study was attempted in an effort to answer the following question: Are some of the pupil characteristics listed above related to mathematics learning in a 'self-directed' setting? In other words, can some of the characteristics included in this study be used to predict success for mathematics learning in such a setting? Of what importance are these variables in settings that are partially teacher-directed or teacher-directed?

III. THE NULL HYPOTHESES

The purposes stated and the questions raised gave rise to the following null-hypotheses:

1. For subjects in the SD-group there exist no significant correlations between the initial learning test scores or retention test scores and the following variables:

   a. intellectual ability

   b. reading ability

   c. socio-economic status

d. ability to make personal adjustment

e. ability to make social adjustment.

2. For the subjects in the SD-group there is no significant difference in mean initial learning test scores or retention scores of pupils who are grouped as being:

a. reflective or impulsive

b. male or female

3. For the subjects in the PTD-group there exist no significant correlations between the initial learning test scores or retention test scores and the following variables:

a. intellectual ability

b. reading ability

c. socio-economic status

d. ability to make personal adjustment

e. ability to make social adjustment

4. For the subjects in the PTD-group there is no significant difference in mean initial learning test scores or retention scores of pupils who are grouped as being:

a. reflective or impulsive

b. male or female

5. For the subjects in the TD-group there exist no significant correlations between the initial learning test or retention test scores and the following variables:

a. intellectual ability

b. reading ability

c. socio-economic status

d. ability to make personal adjustment

e. ability to make social adjustment

6. For the subjects in the TD-group there is no significant difference in mean initial learning test scores or retention scores of pupils who are grouped as being:

a. reflective or impulsive

b. male or female

These hypotheses were tested at the .01 level of significance.

IV. DEFINITION OF TERMS

For the purpose of this study terms were defined as follows:

Self-Directed (SD): Instructions, explanations, and activities were presented to small groups in the form of written material without direct or formal teaching.

Teacher-Directed (TD): The topics were formally introduced to the class by the teacher, examples were presented and individual seatwork was assigned.

Partially Teacher-Directed (PTD): An introduction relevant to the activities was presented by the teacher and then the activities were completed in small groups.

Initial Learning: Based on the results of the constructed three-part test administered upon completion of the study. (Appendix A)

Retention: Based on the results of the constructed three-part test administered four weeks after the study. (Appendix A)

Recognition: Recall of material in the form in which it was presented.

Algorithmic Thinking: Generalization or transfer from learned material to material that is similar to it.

Open Search: Problem solving that is not confined to operations and the

solutions of problems for which a straightforward procedure has been learned.

Intellectual Ability: Based on the scores of the California Short-Form Test of Mental Maturity.

Conceptual Tempo: Impulsiveness-Reflectiveness, based on the results of six questions from the Matching Familiar Figures Test.

Personal - Adjustment: Based on the scores of Part 1 of the California Test of Personality which includes measures of such components as: self-reliance, sense of personal worth, sense of personal freedom, feeling of belonging, freedom from withdrawing tendencies and nervous symptoms. (Appendix B)

Social - Adjustment: Based on the scores of Part 2 of the California Test of Personality which includes measures of such components as: Social standards, social skills, freedom from anti-social tendencies, and family, school and community relations. (Appendix B)

Total - Adjustment: Based on the total score (Parts 1 and 2) of the California Test of Personality.

Socio-Economic Status: Based on a score that was obtained by matching the father's occupation with a scale designed by Blishen (1968).

Reading Ability: Based on the scores of Part 2, the Paragraph Meaning Test, of the Stanford Achievement Test.

Open-Area School: An elementary school which contains an area designed to accommodate several groups of students representing various grade levels. Besides providing space for pupils to meet as a class group, the area usually includes an instructional material center or library and some spaces designed for individual study.

V. SIGNIFICANCE OF THE STUDY

The changes that are suggested for elementary education, and specifically for the mathematics classroom, will put the pupil into a setting that requires a new role. Learning is becoming a more active process and to a great extent more self-directed.

The results of some studies and observations reported by educators and researchers show that there may exist many advantages that can be attributed to such an approach of teaching mathematics. However, many of the discussions about advantages appear to be of a subjective nature.

Justification for this research lies in the fact that little is known about how effective various approaches to the teaching of mathematics are for different learners. To make meaningful allowances for individual differences, answers to the question 'What kind of student is best suited for an approach that is self-directed, or partially-teacher-directed, or teacher-directed?' are necessary.

VI. LIMITATIONS

The samples for this study were selected from a population of grade five pupils in two urban schools. It may be that older or younger children or children from different schools would react differently to the procedure outlined for the three groups. Since the number of mathematical concepts chosen was limited, other concepts may be more or less suitable for this kind of an approach. There might also exist a better or more effective way of preparing and presenting the topics that were used in this study.

The child-centered or self-directed activity approach to the teaching of mathematics is difficult to define or describe and usually examples are used to accomplish this task. (Kaye, 1968; Biggs, E., 1968, 1969). However, one outstanding feature of the approach is that it is 'open-ended'. The 'open-endedness' was somewhat limited to control for discrepancies in time between the three groups and this must be considered in interpreting the results and making generalizations about them. Subjects were permitted to complete 'open-ended' tasks after school if they desired. However, actual class-time was the same for the self-directed, partially-teacher-directed and the teacher-directed groups.

An across-group comparison could not be made since the three treatments were assigned to the groups on the basis of a predetermined criterion. To avoid introducing a totally strange procedure to the subjects in the three groups, an attempt was made to match group membership with past experiences of mathematics learning.

VII. OUTLINE OF THE REPORT

In Chapter II some theoretical aspects and a few empirical enquiries that have contributed to recent trends in elementary school mathematics teaching are discussed.

In Chapter III the samples, teaching methods, instruments, testing procedures and statistical analyses are described.

In Chapter IV the data are analyzed and the findings are presented.

Finally, in the last chapter, the study is summarized and some recommendations for further research are made.

CHAPTER II

SELECTED RELATED LITERATURE

Considered in a theoretical framework the new knowledge that has contributed to recent changes and led to new approaches of teaching elementary mathematics comes from three distinct fields:

1. developmental theory
2. learning theory, and
3. empirical enquiry.

I. DEVELOPMENTAL THEORY

Jean Piaget stands out as the most important man in providing observations and research relevant to the problems of learning mathematics. Piaget's developmental theory and the results of his ingeneous experiments (Piaget, 1952, 1960, 1964, 1967) contain many valuable suggestions as to how children develop and how they learn mathematical concepts. Piaget himself has not written extensively on the educational implications of his developmental theory. He has stated (Piaget, 1951, pp. 95-98) that an individual's apparent failure to grasp the most basic concepts of elementary mathematics stems not from a lack of any special aptitude but rather from affective, emotional blocking or inadequate preparation. Frequent failures of formal education can be traced to the fact that it begins with language, illustrations, and narrated action rather than real practical action.

Concrete experiences of a child are of fundamental importance. The child "must have the actual situation to handle or at least images of these situations" (Dienes, 1966, p. 32). This practical activity should be systematically developed and amplified throughout the primary grades. Piaget also has said:

> The question comes up whether to teach the structure or to present the child with situations where he is active and creates the structure himself ... The goal in education is not to increase the amount of knowledge, but to create the possibilities for a child to invent and discover himself ... Teaching means creating situations where structures can be discovered; it does not mean transmitting structure which may be assimilated at nothing other than a verbal level (Duckworth, 1964, p. 498).

Several authors have attempted to state the important implications of his theory.

Sullivan (1967) shows how Piaget's description of development could be used as an aid in assessment of learning outcomes in a curriculum. Furthermore, he describes how Piaget's observation of various stages could be used as an aid in the structuring and sequencing of subject matter in a curriculum, and how these observations are suggestive of certain types of learning atmospheres. This is why Piaget is more favorably received by persons who are predisposed to "discovery" and "activity" learning approaches as opposed to learning through direct verbal interchange between teacher and child. The theory results in an emphasis on concrete manipulation of his environment by the child.

Harrison (1967) relates Piaget's theory to mathematics education. Concepts, he states, should be built from appropriate "concrete experiences." Students can be led to create their own notions of the structure of a subject if they are given a rich experiential background. Flexible thinking is encouraged by giving students opportunities to

approach problems from several points of view and from different methods of attack. In another article Harrison (1969) shows how Piagetian ideas can relate to mathematics teaching and learning at the secondary level.

Biggs and MacLean (1969) make many valuable suggestions for an active approach to learning mathematics. An attempt has been made by these authors to incorporate two of the points emphasized by Piaget about active learning. First, a child must be allowed to do things over and over again and thus reassure himself that what he has learned is true. Secondly, this practice should be enjoyable. The authors also state that the image of the teacher as "the fountain of all knowledge" must disappear. Teachers should not give or "teach" a lesson, but provide a focus for discussion.

Adler (1966) points out that the mathematical experiences a child is given at any age should be experiences he is ready for in terms of the stage of mental growth he has reached, and they should help prepare him to advance to the next stage. Concepts should be built from appropriate concrete experiences. Manipulating sets of objects may help in gaining a better grasp of relations. In the development of new concepts at any level it is necessary to proceed from the concrete to the abstract. Errors in thinking should be overcome by providing experiences that will expose these errors and enable a child to correct himself. A variety of approaches enable a child to see things from many points of view and in this way mental growth is encouraged.

Almy (1968) concludes that demonstrations and pictured illustrations are less meaningful than activities. Direct experience is the avenue to knowledge and logical ability. The ability to use language to express logic is an outcome of activity. Attempts to improve the child's logic

solely through instructing him in the use of language are not likely to be very successful. Physical activity and social exchanges with his peers develop a child's logical thinking. It is possible that a child learns more readily from peers than from adult instruction. Listening to children may thus be more valuable than telling them.

The above statements imply that meaningful learning cannot take place by merely presenting materials from a printed page such as a textbook. Active involvement, concrete materials, and socialization amongst peers are necessary ingredients for a healthy and efficient learning environment. A setting like this would narrow the gap between real life learning and 'traditional' school learning. Duckworth (1964, p. 497) states that:

> You cannot further understanding in a child simply by talking to him. Good pedagogy must involve presenting the child with situations in which he himself experiments, in the broadest sense of that term - trying things out to see what happens, manipulating things, manipulating symbols, posing questions, and seeking his own answers, reconciling what he finds at one time with what he finds at another, comparing his findings with those of other children.

II. LEARNING THEORY

Dienes (1960, 1963, 1966, 1967, 1970) has developed a theory of mathematics learning and teaching that is based on the assumption of the correctness of Piaget's work. In commenting on some of the basic processes involved in mathematical learning he states that exploring and manipulating should be a part of mathematics lessons. Play should be regarded as an integral part of any learning cycle and mathematical play should be generated by providing children with a large variety of constructed mathematical materials. A large number of different situations

must be provided to enable a child to make abstractions. Mathematical concepts should be taught by changing the values of the mathematical variables and thus encouraging children to generalize. Children, according to Dienes, are able to engage in quite sophisticated logical thinking if the stimulus situations are of a concrete character.

This latter statement seems to support Bruner's (1960) hypothesis that any subject can be taught to any child at any level in some honest form. Bruner also states that it is important to translate mathematics to the child's way of thinking and that basic ideas should be presented in a concrete form as early as possible and that they should be represented and developed in more and more abstract forms as the child's thinking processes mature.

Fehr (1968) defines mathematics as a way of thinking, a medium for communication, a procedure for quantification of problems and a description of observed phenomena. According to Fehr, the approach that is most effective for the learning of mathematics and the one that has the most promise of permanence and of transfer of learning to the solution of quantitative problems is the process of going from experience with things, to thinking about things, to abstraction and concept formation, to thinking with the concept, to symbolization and manipulation, and finally to re-organization of the newly learned concept into the whole structure.

Gagné's (1965, 1967) theory of knowledge acquisition proposes some manner of functioning for learning sets in a hierarchy. He hypothesizes that within this hierarchy specific transfer from one learning set to another set of higher order will be zero if the lower one cannot be recalled. If it can be recalled, transfer will range up to

one-hundred per cent. Such curriculum projects as the Individually Prescribed Instruction (IPI) have been heavily influenced by the learning hierarchy concept. This hierarchy, according to Bolvin (1968), permits the analysis of the behavior required as pre-requisites to a given objective. This analysis serves as a guide in sequencing and ordering objectives and it can be determined which objectives are useful in attaining another objective.

Suppes (1966) is another educator who has contributed to the development of theoretical constructs. One of his models has implications for sequencing in mathematics. He believes that in analyzing performance, latencies are in many respects more important as a source of information to the theorist than response data. Suppes suggests that response latencies may be a good device for determining the nature of the cognitive strategies students use for solving certain classes of problems. The study reported (Suppes and Groen, 1966) indicates how latencies might be employed to provide fruitful information for the study of sequencing.

In summary, the theories and constructs that were briefly mentioned seem to support the principle that learning is a developmental process. As the child explores his environment, mathematical notions arise in his mind and he becomes aware of characteristics and properties of objects as well as relations among ideas. As the child gets older these notions are refined and they result in highly structured concepts. Eventually he is enabled to deal with abstractions and symbols.

Statements on the developmental and learning theories emphasize some significant aspects of mathematics learning for children in the elementary school. Children should be able to manipulate objects or symbols in various ways and they should be given an opportunity to invent, discover, raise questions, compare results and exchange ideas

with their peers. Exploring, playing, and experiencing with things in a variety of situations leads to symbolization of, or thinking with, the concept.

III. RELATED EMPIRICAL ENQUIRIES AND LITERATURE

In part, the preceding theories and statements have led to new approaches in teaching mathematics. Changes on this continent have been slow. In England, for example, changes in classroom procedure began more than thirty years ago and experiments in the teaching of mathematics began about ten years ago. (Biggs, 1968, A). The Nuffield Mathematics Teaching Project, for one, emphasizes learning by doing with its ultimate goal to produce happy children capable of thinking for themselves (Matthews, 1968).

The new methods of teaching mathematics are given various labels. The most common is probably "discovery approach". Regardless of the label, the one common feature present is that there exists the attempt to offer a child-centered activity-oriented program that encourages children to think, investigate, and discover for themselves.

Glennon (1968, pp. 73-75) lists sources that give rise to claims both for and against discovery learning. The list in favor includes: natural and preferred way of learning, motivating power, and better learning and retention. Counter claims include: increased expenditure of time, possible frustration, not suited for all children, and at certain age levels understanding is more general, clearer and more precise when learned through verbal presentation.

In his article Bittinger (1968) reviews studies that dealt with discovery and concludes that "the studies do not seem definitely conclusive in view of their possible contradictions. Many of them fail to be convincing because they lack rationality of learning or because they fail to give the didactic method a fair chance" (p. 144). He reports one classroom oriented study that was conducted by Suchman at the elementary level in which it was found that after twenty-four weeks the subjects in only two out of six schools performed significantly higher on a test of content. He also found that inquiry trained children were more fluent; they asked fifty per cent more questions about the test problems; and they asked more analytical questions. However, there was no significant difference on tests which measured such things as principles underlying the demonstrated phenomena, necessary events occurring in problem solving, and importance of related parameters.

Kieren (1969) reviewed recent research on activity learning under the headings of: Discovery Teaching and Learning in Mathematics, and Manipulative Learning in Mathematics. He reports that the literature on discovery suggests a conflict between methodologies which will sponsor maximum understanding and those which sponsor maximum motivation to continued learning. He concludes that recent research (1964-1968) does little to resolve this theoretical conflict. Kieren agrees with Bittinger by stating that the results present a mixed picture and that the discovery methodology used is frequently not clearly defined. Kieren also questions the quality of the research done on manipulative play and he concludes that many of the studies are pilot studies in some sense and often the questions asked were too simple. Careful definition of the problem is frequently lacking.

One study which involved elementary school children (grades five and six) was conducted by Worthen (1967). He made use of two different treatments. For one method, verbalization of each concept or generalization was delayed until the end of the instructional sequence (discovery method), and for the other method (expository), verbalization of each concept or generalization was the initial step in the instructional sequence. Two classes in each of eight schools served as experimental groups, and the subjects in the two classrooms were instructed by the same selected teacher. The mathematical concepts were selected on the basis of suitability for both discovery and expository teaching and probable unfamiliarity to subjects at the inception of the study and included: notation, addition, and multiplication of integers; the distributive principle of multiplication over addition; and exponential notation and multiplication and division of numbers expressed in exponential notation.

Worthen found that the pupils taught by the discovery method were able to retain significantly more material after a five week delay and again eleven weeks after the instructions. These subjects also showed superior ability to use discovery problem-solving approaches in new situations whether they require paper and pencil application or involve verbal presentation by the teacher. However, there appear to be a few weaknesses in the study which could lead to systematic bias and thus contaminate the results. Even though the teachers were trained, one criterion for their selection was willingness to participate in the research project. Also the same teacher taught both approaches or treatments in each of the schools. This would make it rather difficult to assess the role expectations played in obtaining the results.

Shuster and Pigge (1965) used different teaching methods by varying the amount of developmental-meaningful activities and drill activities according to the following percentages: 75 - 25; 50 - 50; and 25 - 75 respectively. Their results led them to conclude that children learned skills better by spending less time on drill and more time on developmental-meaningful activities.

Tanner (1969) reviewed various studies and classified them according to mathematics, science, rules of coding and classifying, and miscellaneous. He concludes that the studies and results defy synthesis. Not only do they present contradictory and non-significant results, they also were conceptualized in a great variety of ways when such things as method, time and medium of instruction are considered. Tanner concludes that a satisfactory synthesis is not possible as yet and experimental studies of 'discovery learning' do not provide sufficient rationale for sweeping changes in curriculum and instruction. One of his suggestions for directions in which further studies might proceed is to attempt to investigate the effectiveness of different treatments for different learners.

Numerous advantages of the 'activity' or experience approach are discussed by different authors. However, some of these tend to be very subjective in nature and others seemed to be based on casual observation. Bruner (Downey, 1965, p. 74) suggests four benefits: "The increase in intellectual potency; the shift from extrinsic to intrinsic rewards; the learning of the heuristics of discovering; and the aid to conserving memory."

Ausubel (Saylor and Alexander, 1966, pp. 185 - 186) believes that discovery methods are useful in the early, unsophisticated stages of the learning of any abstract subject matter and that various cognitive and motivational factors undoubtedly enhance the learning, retention, and transferability of meaningful material. He also points out that many advocated advantages of the discovery approach, including those stated by Bruner, lack supporting research data.

Biggs (1968-A) discusses various advantages of the activity approach. Pupils are allowed to think for themselves. They can investigate mathematical problems in individual and original ways. Appropriate activities stimulate the student's curiosity and children acquire skills from devising and refining their own methods for written calculations. The approach allows for independent discovery which in turn leads to satisfaction and success, and working in groups enables the children to learn from each other. Additional statements include such generalizations as: catches the child's imagination and sustains his enthusiasm; satisfaction of success (Biggs, 1968, A, pp. 2-3); the pupils are relaxed, happy and respectful. The British teachers are equally relaxed, tolerant, and genuinely respectful towards the pupils, (Unkel, 1968, p. 137), and the Chinese proverb, "I hear and I forget; I see and I remember; I do, and I understand", is often quoted to state the advantages of such an approach.

Kaye (1968), who discusses mainly the new role for the teacher, also claims that if the pupil is presented with some investigation that interests him, he will gain arithmetic skill and competence much faster, but probably at a later age than the pupils of the past. The approach creates interest and the emphasis in the classroom changes from

"teaching" to "learning". The noise level in the classroom is high, but discipline problems are minimized. A highly student-centered experience approach leads to more and faster learning than we would expect, and students develop a willingness to tackle new problems.

Servais (1968) who talks about new methods in mathematics education states that"rather than that the students should be disgusted by a routine forced-feeding of ideas imposed from outside, they should find pleasure in searching their way through a situation that awakens their curiosity and challenges their intelligence" (p. 797). The approach which is based on involvement assures adequate motivation. Students learn by experience to schematize, to untangle structure, to define, to demonstrate, to apply themselves instead of listening to ready-made results.

Mathex Bulletins (Encyclopaedia Britannica) provide teachers and students with mathematical topics that are based on the activity or experience approach. Various articles mention advantages of the experiment approach to mathematics. Children observe, invent and thus remember (Sawyer, 1967). The activities are flexible and as each child can make a contribution and this reinforces confidence in himself. Many activities relate mathematics to the experience of the children and in this way the subject comes alive, becomes more meaningful, and generates interest and enthusiasm (MacLean, 1967). The activities permit children to discover important mathematical principles and relationships and the arrangements of these activities develop strategies for thinking about mathematics in general. That is, children learn how to learn (Nelson, 1967). The activities provided for the pupils are meant to lead to a mastery of mathematical calculations but in a degree appropriate to the ability of each pupil (Sawyer, 1968).

It is difficult to summarize the role of 'discovery' teaching since there is so much divergence in the way the term is defined. It seems that pupils who are exposed to an approach that differs from an expository method ask more questions and the questions are more analytical (Bittinger, 1968). Discovery seems to aid retention (Worthen, 1967) and when a greater proportion of time is spent on developmental activities, achievement is higher (Shuster and Pigge, 1965). Other possible advantages include: favorable motivational factors, a shift from extrinsic to intrinsic motivation, stimulation of curiosity, attainment of satisfaction and success, minimized discipline problems, and willingness on behalf of the students to tackle new problems.

IV. VARIABLES AND MATHEMATICS LEARNING

A number of variables will be examined in this study. Some of these variables have been included because other studies have found them to be related to mathematics learning and achievement. Others were included because questions have been raised about the effectiveness of different treatments for different learners (Cathcart, 1969; Tanner, 1969; Vance, 1969), and variables related to personal and social adjustment could be important factors that affect a student's ability to learn by a method which is to a large extent self-directed and group-activity oriented.

Goodnow and Bethon (1966) contend that among school children all tasks show a close relation to mental age. Kennedy and Walsh (Glennon, 1968, p. 42) conclude from their factor-analytic study that as far as intellectual variables are concerned, there is evidence of a strong

factor relating to high achievement and high ability which would seem to indicate that mathematical ability is not a specific ability, but relates to overall high ability. However, Glennon (1968, p. 43) warns that since intelligence tests tend to discriminate against the child from the lower socio-economic class extra care should be used in not excluding a child who is talented but whose measured intelligence scores do not clearly indicate his talent.

Glennon (1968, p. 65) states that "where stress is put on meaningfulness in learning as well as individual discovery of some of the material to be learned, it seems imperative that the student be able to read the textbook(s) with a high degree of competence and confidence." This would be especially the case where some part of the mathematics program is self-directed. The explanations, problems and activities are presented on a printed page and understanding the problem necessitates such reading skills as comprehension of statements and selection of relevant details. These reading skills are essential for selecting the correct procedure to solve problems.

Romberg (1969, p. 480) states that since mathematics has its own symbolism and syntactics, it requires its own reading skill. He reports a study by Call and Wiggin who found that a ten-day unit on the reading of mathematics helped students to solve arithmetic problems. Also mentioned is a study by Gilmary who 'surprisingly' found that remedial reading instruction had a positive effect on arithmetical computation achievement.

Kagan (1963, 1965, 1966) developed a testing procedure that enables an examiner to identify different conceptual tempos. Impulsive and reflective students can be identified in terms of the time they take to respond to various stimuli presented to them in the form of pictures. Kagan found that differences in conceptual tempo might exist even if children are of equal intelligence and his research results suggest that "an individual's preferred conceptual strategy is implicated in a wide variety of behaviors" (1963, p. 109), and it remains the preferred mode over a number of years.

Kagan found that an impulsive child will tend to report the first hypothesis that occurs to him and this response is often incorrect. The reflective child, on the other hand, delays a relatively long time before reporting a solution and is usually correct. He points out that in arithmetic a child is required to make inferences, and certainly any program that emphasizes the discovery method of instruction has as its key aim the practice of inference. Discovery tasks place an impulsive answerer at a disadvantage since he is wrong more frequently and thus experiences more negative reinforcement.

Cathcart and Liedtke (1969) found that reflective children in grade two scored significantly higher on an achievement test that consisted of three parts: concepts and their application, ability to solve verbal problems and ability to recall basic facts.

Glennon (1968, pp. 47-48) reviews several studies on the effect of 'cultural deprivation' on achievement in mathematics. He concludes that (p. 48),

> the research evidence tends to indicate that where no specific intervention in the education of the culturally disadvantaged child takes place, the child will deviate negatively from the middle class achievement norm in arithmetic.

This deviation tends to become greater as the student progresses through school. There exists, however, much evidence that points to a great variance in achievement demonstrated by 'culturally deprived' children.

Jarvis (Glennon, 1968, p. 49) included 350 boys and 350 girls of similar chronological age in a study to determine whether there exist differences in achievement in elementary school mathematics (grade six) between these two groups. The results led him to conclude that bright boys are superior to their peer group girls in both reasoning and fundamentals. Male students showed more ability than the female students to perform arithmetic reasoning functions. However, with the exception of the 'bright' group, girls were superior to boys in their ability to execute the arithmetic fundamental operations. Findings of a similar study lead Glennon (p. 50) to the conclusion that when either extreme of the I.Q. range is excluded boys will achieve higher than girls on tests dealing with mathematical reasoning while the girls will achieve higher on tests on computational ability.

Semler (1960) examined as part of her study the relationship between personality characteristics as measured on the California Test of Personality (CTP) and academic achievement. She used 438 fifth-grade subjects and found a correlation of .32 between personality and achievement scores which were obtained from each subject's cummulative record.

In his survey of related research, Neufeld (1967) reviews several studies that deal with measures of personality. He points out that many investigators have found positive relationships between personality factors and achievement under conventional instructional methods. However, recent studies involving programmed materials have reported negative relationships. As part of his study Neufeld used the sub-test scores of the CTP to investigate the relationship between components of personality and achievement gain in elementary school mathematics classes using the Individualized Mathematics Curriculum Project approach. He found that there existed some significant differences in personality characteristics among pupils of different levels of mathematical achievement gain when such variables as sense of personal worth, social standards, freedom from withdrawal tendencies, social skills, freedom from anti-social tendencies and community relations were considered.

V. SUMMARY

Piaget's theory of intellectual development describes learning as orginating from the child and his own interests and drives. The child learns from active interaction with his environment and doing things that are interesting to him. He explores, experiments, and modifies his behavior and conceptions of the world by means of self-fulfilling drives.

During the first five or six years of his school-life, concepts such as space, time, relations, classes and combinations become available to the child.

> ... these general concepts are the stuff of general knowledge and intelligence. These general concepts of the developing intelligence evolve whether the child goes to school or not because they are not dependent upon specific instruction (Furth, 1970, p. 3).

Piaget's theory, when applied to the elementary school, is suggestive of certain types of learning atmospheres which, according to various authors, would narrow the gap between real life learning and traditional school learning. Duckworth (1964), Sullivan (1967), Harrison (1967, 1969), Biggs and MacLean (1969), Adler (1966) and Almy (1968) have made suggestions which are applicable to the mathematics classroom in the elementary school. These include: an active involvement by the learner; manipulation of objects and symbols by the learner; working in a peer group; time by the learner to pose questions, experiment, and seek his own answers and the possibility to compare his results with other children.

Dienes (1966, 1967) and Fehr (1968) have developed models of mathematics learning which are based on the assumption of the correctness of Piaget's work. Incorporated into their models are most of the suggestions listed above. Dienes and Fehr show how these suggestions can be used to stimulate the development of the ability of making abstractions and the learning of mathematical concepts.

Results from various studies seem to suggest that there exist certain advantages for young children when they are presented with an approach that differs from the expository method. Some advantages of an activity or discovery approach include: better learning and retention (Worthen, 1967; Shuster, 1965); increased fluency in asking questions (Worthen, 1967); high motivating power and sustained enthusiasm (Biggs, 1968 - A); willingness to tackle new problems (MacLean, 1967); attainment of satisfaction and success (Biggs, 1968 - A); and minimized

discipline problems (Kaye, 1968). Some counter claims (Worthen, 1967; Ausubel, 1967) include: an increased expenditure of time; possible frustration; unsuited for all learners; and at certain age levels understanding is more general, clearer and more precise when learned through verbal presentation.

The variables which have been shown to be positively related to mathematics achievement include intelligence (Glennon, 1968), reading ability (Glennon, 1968 ; Romberg, 1969) and socio-economic status (Glennon, 1968). There appear to be differences between boys and girls when mathematical reasoning and computational ability are considered (Glennon, 1968).

Neufeld (1967) points out that there exists some evidence to suggest that the relationship between personality factors and mathematics achievement might vary for different instructional methods. Kagan (1963) suggests that an individual's preferred conceptual strategy is implicated in a variety of behaviors and Cathcart (1969) found that children classified as reflective scored higher on an achievement test than impulsive children.

The purpose of the present study was to examine the relationship of the variables mentioned above to mathematics learning in settings which were self-directed, partially teacher-directed and teacher-directed.

CHAPTER III

THE EXPERIMENTAL DESIGN

This chapter contains an explanation of the design that was employed and a description of the materials that were used. The pilot study and the samples are discussed. A description of the tests that were used, the testing procedure employed, and the scoring and statistical methods used to analyze the data of the study is also included.

I. DESIGN AND MATERIALS

The major purpose of this study was to examine the role of certain pupil characteristics in three different settings. For the self-directed (SD) setting an attempt was made to minimize the direct role of the teacher. His main responsibility was to distribute the materials and to answer questions that were asked by individuals as they worked in groups and attempted to solve the problems presented. Any checking of answers was voluntary. The groups were asked to reach a consensus on their answers. The correctness of their replies could be checked from the blackboard the following session.

The characteristics that were considered in this study included levels of: intelligence, reading ability, conceptual tempo, ability to make personal adjustment, ability to make social adjustment, sex and socio-economic status. In order to determine which of these variables could be used to predict learning outcomes for a teacher-directed and a partially teacher-directed setting, two other treatments

were considered. One of these treatments was very teacher-directed (TD). In this setting the teacher introduced the topics and assigned work to be completed by the individual members of the group. The checking of the answers was done as a group activity at the beginning of the following session. For the other treatment (PTD), the teacher introduced the topics, but then the pupils worked in groups while attempting to solve the problems. The answers to the problems were written on the board the following day but the checking of these was not mandatory. Material was prepared for approximately four weeks of class periods that lasted about fifty-five minutes. The main topics covered included decimal fractions, number pairs, perimeter, area, number arrays, maps (contours and scale), curves, region, networks, multiplication and probability (Appendix C).

A pilot study was conducted to check on the suitability of the content and the approximate time the students would take to complete the activities. Following this some revisions were made and some of the activities included were made optional since it took these subjects longer than expected to complete the exercises. The optional activities are indicated with an asterisk in Appendix C.

Only a few minor differences existed between the material for the three treatments. The SD- and the PTD-groups received the material as it is shown in Appendix C. Only a few minor changes were made for the TD-group. These included the rewriting of six pages (Appendix D) and the leaving out of instructions that were directed specifically toward group activities or activities that involved working with a partner.

Each subject received a copy of the activity sheets. The teachers' copies were prepared in booklet form. These booklets contained

the topics, a division of the topics into lessons, objectives for each lesson, answers to all the exercises, a list of the materials that would be needed and a list of review questions.

II. INSTRUMENTATION

In this section each of the instruments used to test the hypotheses stated in Chapter I is described.

Matching Familiar Figures Test: This instrument is a modification of a test designed by Jerome Kagan (1965) and it enables an examiner to identify different conceptual tempos in pupils. Each item on this test consisted of pictures on two 8½ by 11 inch sheets of paper. On one page a picture was presented as a standard. The other page contained six pictures similar to the standard. However, only one of the six was exactly like the standard. As a subject faced the two pages the following directions were given: "________(name of subject), I am going to show you a picture of something you know and then some pictures that look like it. I would like you to find the picture on this page (point to page with six pictures) that is most like the picture on this page (point to single picture)." If the response given by the child was correct a simple comment like, "Good, let's try another one" was made. In the case of an incorrect response the comment consisted of "No, there is one closer than that. Try to find it." The time taken until the first response was given was measured with a stop watch to the nearest half second and a record of each response was kept until the correct answer was given. Thus impulsive (fast) and reflective (slow) students were identified in terms of the time taken by each one to respond to

the various stimuli presented to them and in terms of the number of incorrect responses each one gave. A median split on both average errors and average time was used to classify the subjects according to conceptual tempo.

The whole test consists of two practice items and twelve test items. Prior to the study this test was given to ten subjects to obtain an estimate of the administration time. It took between fifteen and twenty minutes for each individual. A further check revealed that little information was lost by cutting the test down to the first six test-items. Thus, the first six items were used as part of this study and it took about ten minutes to test each subject individually.

California Test of Personality: This test designed by Thorpe, Clark, and Tiegs (1953) is organized around the concept of life adjustment as a balance between personal and social adjustment. Personal adjustment is assumed to be based on feelings of personal security and social adjustment on feelings of social security. The test consists of twelve subtests (defined in Appendix B), each containing twelve items. One half of the subtests and items are designed to measure components of personal adjustment, whereas the remaining subtests and items measure components of social security. According to the authors the subtests or components are not names for so-called general traits but rather names for groupings of more or less specific tendencies to feel, think and act.

Sims (Buros, 1959) concludes that "as a measure of self-concept in the, as of now, vaguely defined area called adjustment, this test is as valid as most such instruments" (p. 39). This conclusion is based on the fact that the items of this test are based on research results and

were constructed with the help of counselors, clinical psychologists and teachers.

Reliability coefficients (Kuder-Richardson formula) of .94, .93, and .92 are reported by the authors for the whole test, the personal adjustment part, and the social adjustment part, respectively. The reliability coefficients for the subtests range from .59 to .83. Sim's (Buros, 1959) comment on 'fair' reliabilities for the subtests is defended by the authors, and they state that since many of the items touch relatively sensitive personal and social areas and since such student attitudes may change in a relatively short time, the statistical reliability of instruments of this type will sometimes appear to be somewhat lower than that of good tests of ability and achievement. The intercorrelations between subtests range from .22 to .59 and appear low enough to warrant the use of each subtest as a separate measure.

Sims concludes his review of the test by stating that "... as personality inventories go, the California test would appear to be among the better ones available" (p. 39).

<u>California Short-Form Test of Mental Maturity</u>: This test (Sullivan, et. al. 1961) is described by the authors as "an instrument for appraising mental development or 'mental capacity'." It contains seven subtests which sample four main areas of mental activity (termed "mental factors"): spatial relations, logical reasoning, numerical reasoning and verbal concepts. These four factors are covered in both, a language and a non-language section of the test. The results of the test yield an I.Q. score for the whole test as well as scores for the language and non-language components.

Burt (Buros, 1959, p. 313) in his review of the test reports that the reliability (K-R21) for the total scores varies between .87 and .89 at most grade levels. Validity coefficients based on observed and corrected correlations with the Stanford-Binet, WISC and with other group intelligence tests are reported to average about .75, "but correlations of this nature are not very informative" (p. 314).

The intercorrelations between the measurements for the four "mental factors" range from .30 to .60 and these support the authors' claim (according to Burt) that the test taken as a whole provides an excellent instrument for assessing general 'capacity'. However, Burt questions the value of some of the subtests as reliable and valid assessments of so-called special factors, since they only take from seven to fifteen minutes to complete. Low reliabilities which range from .50 to .75 appear to support his doubt.

Burt also states that the names given to the factors and the detailed instructions for their measurement might encourage the examiner to extract far more information from the test results than is actually present. "These minor criticisms, however, in no way affect the general merits of the test as a whole" (p. 314).

Socio-Economic Status: To determine a measure associated with this variable, Blishen's revised Occupational Class Scale was used. Blishen used data on education and income from the 1951 Canadian census to construct his first ranking of occupations. He used these same variables and additional information to construct his revised scale which is based on the 1961 census data. Blishen reports a rank correlation of .96 between the two scales.

For this study, the occupation of the father was obtained from the school records. If the father's occupation was not available, the mother's occupation was used instead. The occupations were then located on Blishen's socio-economic index and a number was assigned to them by two different persons. The results were compared and in case of disagreement additional information was obtained from the child concerned and the above procedure was repeated.

Stanford-Achievement Test - Reading: The Intermediate I Battery (Kelley, et. al., 1964) includes ten tests. Only one of these, the Paragraph Meaning Test, was used with the subjects in this study. This test consists of paragraphs which contain blanks. The pupil is required to select words or phrases that best fit the blanks.

Robinson (Buros, 1959, p. 656) feels that such a test format favors pupils who have had considerable experience and instruction in using context clues. They could earn higher scores, he claims, even though they can read less well than other pupils who have had no such instruction. Robinson also points out that this technique of testing limits the range of comprehension abilities which can be measured.

The test authors' claim for validity of the test is based on their having made an examination of appropriate courses of study and textbooks in determining the skills, knowledge and understandings to be measured.

The reported reliabilities of the Paragraph Meaning Test are .92 (split-half) and .91 (K-R20).

Mathematics Pre-Test: This test consisted of twenty multiple choice questions. It was constructed by the investigator and administered to all of the subjects before the study began. A copy of the test can be found in Appendix E.

The main purpose of this study was to investigate the relationship between certain pupil characteristics and mathematics achievement in three different settings. The pretest was used to check whether or not there existed any relationships between the variables associated with the pupil characteristics and mathematics knowledge before the treatment began.

The items of the test were based on most of the concepts that were presented to the subjects during the study. Since amount of prior knowledge has been shown to relate to achievement under different learning conditions (Nuthall, 1968), the scores of this test were used to check on the possible existence of such a relationship in each of the three treatments.

The reliability of the pre-test was .60 (K-R20).

Initial Learning and Retention Test: This criterion test consisted of thirty multiple choice questions and it was administered one day after the study was completed. The same test was again administered to the subjects four weeks after the completion of the study to provide a measure of retention. A copy of the test is given in Appendix A.

The questions of the test were based on the material that was presented to the subjects during the study. An attempt was made to include three different types of questions. Part A of the test, which included thirteen questions, dealt with recall of the material as it was presented during the study. Part B consisted of ten questions that required the subjects to apply some generalizations about the material that was presented

to them during the study to material that was very similar to it. Part C, which consisted of seven questions, required the solving of problems that were not confined to operations and solutions of problems for which a straightforward procedure had been presented during the study.

The reliability for the initial learning test was .58 (K-R20) and the correlations between the initial learning and retention test for the three groups ranged from .64 to .74.

III. THE SAMPLES

Prior to the main study a pilot study was carried out. One grade five class from an open-area school in Edmonton participated. This class was not used in the main study. The main purpose of the pilot study was to observe the children in action and to note their reactions to the material presented to them. Results of the observations and the teacher's comments were used to change various parts of the material and some parts of the activity booklet were made optional.

The design of the study required the participation of different groups of students for each of the treatments (SD, PTD, TD). When treatments such as these are used, the teacher variable becomes a very dominant factor. To control in some way for this factor it was planned to select groups of students from open-area schools. In this way two classes of students could be combined for the duration of the study and the influence of any one teacher could be limited by having him work with another teacher as a team. Since two of the treatments (PTD and TD) required the formal introduction of a topic, the teachers could then be asked to take turns with this task.

A request for six grade-five classes from open-area schools was made to the Edmonton Public School Board. Since the method of teaching or presenting the prepared material for one of the treatments (SD) differed somewhat from the textbook oriented mode to teaching mathematics, an attempt was made to obtain a school where such an approach had been used as part of the mathematics program. This was considered important since the study would only last for about four weeks and a new approach would use up a great portion of this time to acquaint the pupils with it.

The reply to the above mentioned request included the names of three open-area schools. The principals of these schools were contacted and after talking to the teachers, the researcher selected two of the schools for the study. Four classes of grade five pupils came from one school (School-1) and one and one-half classes from the other (School-2).

School-1 had an open-area as part of the building. However, the majority of the classrooms in this school were of the conventional type. During the day various classes moved in and out of the instructional area. Two grade five classes were located side-by-side in the open-area. Part of their program was group-activity oriented and part of their mathematics program included activities, games, and exercises that were not based on any one textbook. These two classes were selected to be part of the SD-treatment.

Two other classes in School-1, whose home area consisted of a conventional classroom but who moved into the open-area at the same time for part of their instructions, were selected for the PTD-treatment. Their past mathematics experience was textbook oriented.

All the subjects in School-2 were part of the TD-treatment. Some of these subjects shared an instructional area with a group of grade four

pupils. However, for the duration of the study all the subjects worked together in one area. In this school, mathematics teaching was based on the textbook.

The number of subjects in each of the schools and involved in each of the treatments who completed the study are summarized in Table I. Seven subjects were excluded from the study because they missed more than one-fourth of the total number of days during the study and one subject (PTD) was dropped because he failed to complete any of the activities or tests assigned to him. Of the eight subjects that were excluded from the study, five were from the SD-group, two from the PTD-group and one from the TD-group.

TABLE I

DISTRIBUTION OF SUBJECTS FOR THE TREATMENTS

School	Treatment	Boys	Girls	Total
1	SD	30	21	51
	PTD	32	21	53
2	TD	16	21	37

IV. PROCEDURE AND DATA ANALYSIS

The study was conducted during the months of March and April, 1970. Prior to the study, a meeting was held with the participating teachers. The purpose of the study was explained to them, the materials

and the general procedure were discussed, and questions were answered. The teachers in charge of the SD-and PTD-treatments felt that it would be best if students were assigned to the instructional groups of four at the beginning of the study. Some interchange within groups would be permitted and any subject wanting to work by himself was allowed to do so.

While the study was in progress, the researcher observed the pupils in action on various occasions. Some of their comments and reactions were noted and their work was observed. Attention was paid to such things as method of attacking the problem, mobility within each group, mobility between groups, and the teacher's role.

The standardized group tests were administered by the experimenter during the first two weeks of the study. The individual testing for the Matching - Familiar - Figures Test began during the study and was continued for about one week after its completion. All the tests were marked by the experimenter. Before the results were recorded on a data punching form, random re-checks of the tests were carried out.

All the data were analyzed by the University computer. Programs supplied and documented by the Division of Educational Research (1969) of the Faculty of Education, University of Alberta were used. Some parts of the analysis were done with the aid of a computer program for IBM/67 that uses Iverson's APL-notation.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

In this chapter the findings of the study are presented. The tests of the hypotheses and other analyses of the data are reported. The results for each of the treatments (SD, PTD, TD) are summarized separately and the various hypotheses are analyzed.

I. THE SELF-DIRECTED TREATMENT (SD)

The pretest (Appendix E) consisted of twenty multiple choice questions with four choices for each of the questions. It was administered to the fifty-six SD-subjects one day before the study began.

The test was used as a check on the subject's knowledge of the topics that were presented during the study and the results are summarized in Table II and Figure 1.

TABLE II

PRE-TEST RESULTS FOR SD-SUBJECTS

Range	Mean	Standard Deviation
1-12	5.96	2.77

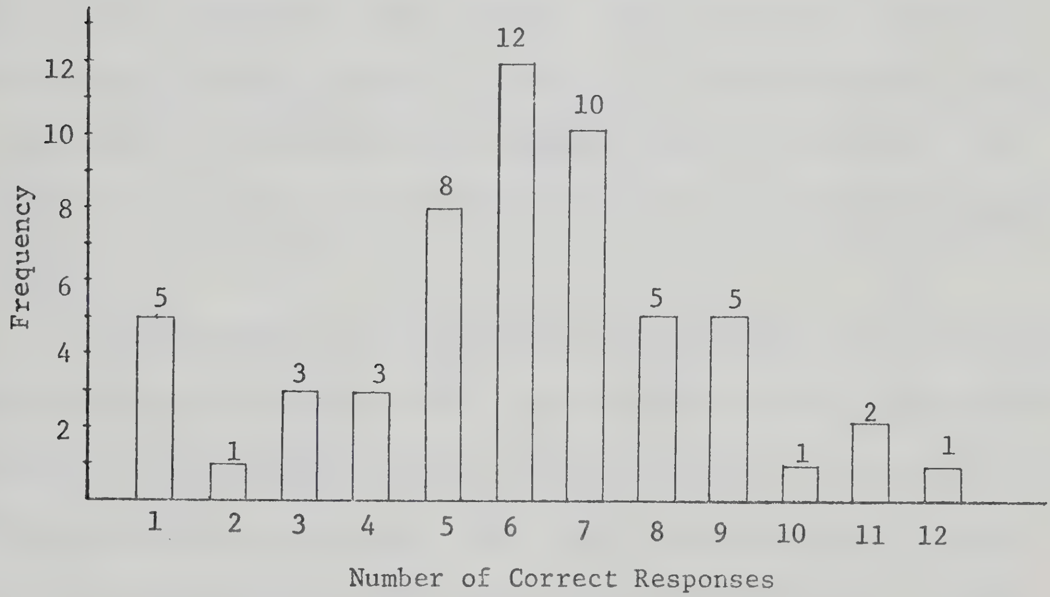

Figure 1

FREQUENCY DISTRIBUTION OF CORRECT RESPONSES ON PRETEST FOR SD-SUBJECTS

During the study, which lasted four weeks, the subjects worked in an open-area. The two teachers in charge of this group felt that in order to avoid confusion it would be best to assign the students to groups before they began to work on the material. The teachers divided the students into groups of four and they attempted to place at least one 'high' achiever into each of the groups. The subjects were told that they could exchange places with other students if they so desired, and

anyone wanting to work by himself was permitted to do so. After the first week, some of the high achievers had moved together and by the end of the study about one-half of the subjects belonged to a different group. Three, and at times four subjects preferred to work alone throughout the study.

The materials for the activities (Appendix C) were placed at one side of the open-area and the appropriate activity sheets were handed out at the beginning of each session. The students were allowed to take sheets beyond those assigned for the day, if they so desired. Only on a few occasions did some of the students take an extra sheet. The completed sheets were filed by the subjects into a loose-leaf binder or notebook.

The students were encouraged to work together as a group and they were asked to discuss their approaches to the activities and the answers. The two teachers were available during each session to give help or to make suggestions. They helped settle arguments and assisted in clarifying problems, but direct answers were not given. At the beginning of each session one of the teachers recorded the answers from the previous day's activities on a blackboard and the students could check their work if they wanted to. The number of subjects who did check their work gradually decreased to about one-fourth.

The noise level for this group could be described as very high, especially at the beginning of each session. It decreased somewhat as each period progressed. At any particular time many of the subjects could be found standing by their desks or kneeling on their chairs while talking to the other members of the group. Frequently some subjects moved to other groups, exchanged ideas, observed, and returned to their tables.

To obtain a measure of intellectual ability, the ability to make personal and social adjustment, and reading ability, the following group-tests were administered: the California Short-Form Test of Mental Maturity, the California Test of Personality, and the Stanford Achievement Test - Reading (Paragraph Meaning). The tests were marked and some of the results are summarized in Table III.

TABLE III

RANGES, MEANS AND STANDARD DEVIATIONS OF INTELLIGENCE -, PERSONALITY - AND READING TEST SCORES FOR SD-SUBJECTS

Test	Range	Mean	Standard Deviation
Intelligence	70 - 134	106.90	12.39
Personality Adjustment			
Total	60 - 133	96.59	20.00
Personal	25 - 65	46.82	10.97
Social	28 - 70	49.76	10.44
Reading	12 - 56	34.45	8.97

Blishen's (1968) revised Occupational Class Scale was used to obtain a measure for each subject's socio-economic status. The appropriate index value was assigned to each subject on the basis of the father's occupation. For three cases the occupation of the mother

had to be used. The range, mean and standard deviation for the index value are shown in Table IV.

TABLE IV

RANGE, MEAN AND STANDARD DEVIATION OF BLISHEN OCCUPATIONAL CLASS SCALE VALUES FOR SD-SUBJECTS

Range	Mean	Standard Deviation
27.0 - 74.3	42.0	10.8

To identify each subject's conceptual tempo, the Matching Familiar Figures (MFF) Test was administered to each subject individually. The time each subject took to give his first response was measured with a stop watch to the nearest half second. A record was also kept of each response until the correct answer was given. The number of incorrect responses was totalled and the average error per question was calculated. The results for the MFF test are tabulated in Table V.

TABLE V

RANGES, MEANS AND STANDARD DEVIATIONS OF MFF-TEST RESULTS FOR SD-SUBJECTS

	Range	Mean	Standard Deviation
Time	8 - 98	26.17	14.79
Errors	.16 - 2.50	.89	.47

To identify impulsive and reflective subjects a scatter diagram of the average response time against the average number of errors was plotted. A division was made to maximize the number of subjects who could be classified as impulsive or reflective. On this basis, an impulsive subject was defined as a subject whose average response time was less than 20 seconds and who averaged more than .83 errors per question. A reflective individual was defined as one whose average response time was more than 20 seconds and who averaged less than 1.00 errors per question. As a result of these divisions, 17 subjects were classified as impulsive and 18 subjects as reflective, as shown in the scatter diagram of Figure 2.

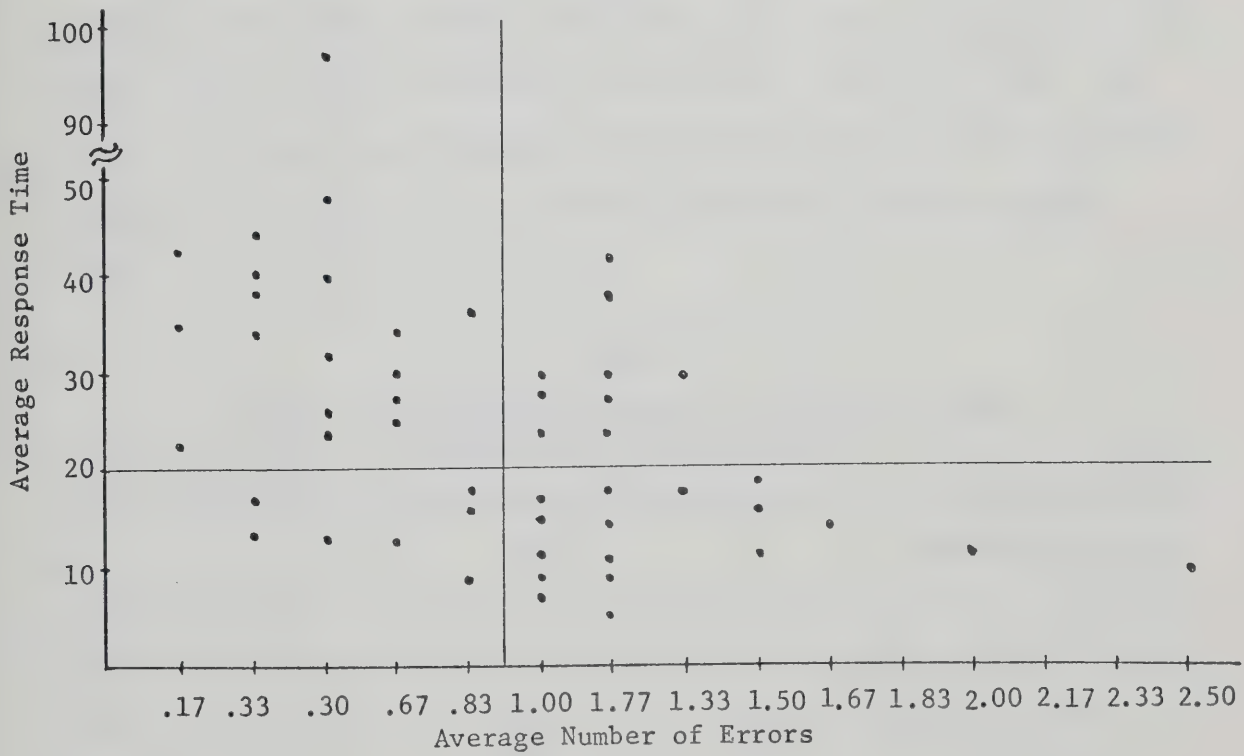

Figure 2

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN TIME AND ERRORS
ON MFF-TEST FOR SD-SUBJECTS

To evaluate the learning outcomes, a three-part test consisting of thirty multiple choice questions was constructed. The thirteen questions in Part A dealt with recall of the material that was presented during the study. Part B of the test consisted of ten questions that required the students to make some generalizations about the topics that were presented to them. Part C was made up of seven questions which involved the solving of problems.

The test was administered one day after the study was completed (Initial Learning Test) and again four weeks later (Retention Test). The subjects were encouraged to attempt all of the questions. In case of uncertainty about the method of solving a problem they were asked to mark the answer they thought to be the correct one. The Initial Learning Test scores ranged from seven to nineteen with a mean of 11.55. The results of the Initial Learning Test are summarized in Table VI and the frequency distribution of the correct responses is shown in Figure 3.

TABLE VI

MEANS AND STANDARD DEVIATIONS OF INITIAL LEARNING TEST SCORES FOR SD-SUBJECTS

	Mean	Standard Deviation
Part A	5.24	2.03
Part B	3.47	1.37
Part C	2.84	1.01
Total	11.55	2.83

For the four weeks between the administration of the Initial Learning Test and the Retention Test the subjects returned to their

regular mathematics program. The participating teachers were asked not to continue with any of the activities that were provided during the study, and to exclude similar topics from their course of study for these four weeks. Fifty of the subjects wrote the Retention Test. The scores for the test ranged from 8 to 21 and the mean was 12.24. The results for the different parts of the Retention Test and the total test are shown in Table VII and the frequency distribution of the correct responses on this test is given in Figure 3.

TABLE VII

MEANS AND STANDARD DEVIATIONS OF RETENTION TEST SCORES FOR SD-SUBJECTS

	Mean	Standard Deviation
Part A	5.88	2.19
Part B	3.42	1.49
Part C	2.92	0.92
Total	12.24	2.83

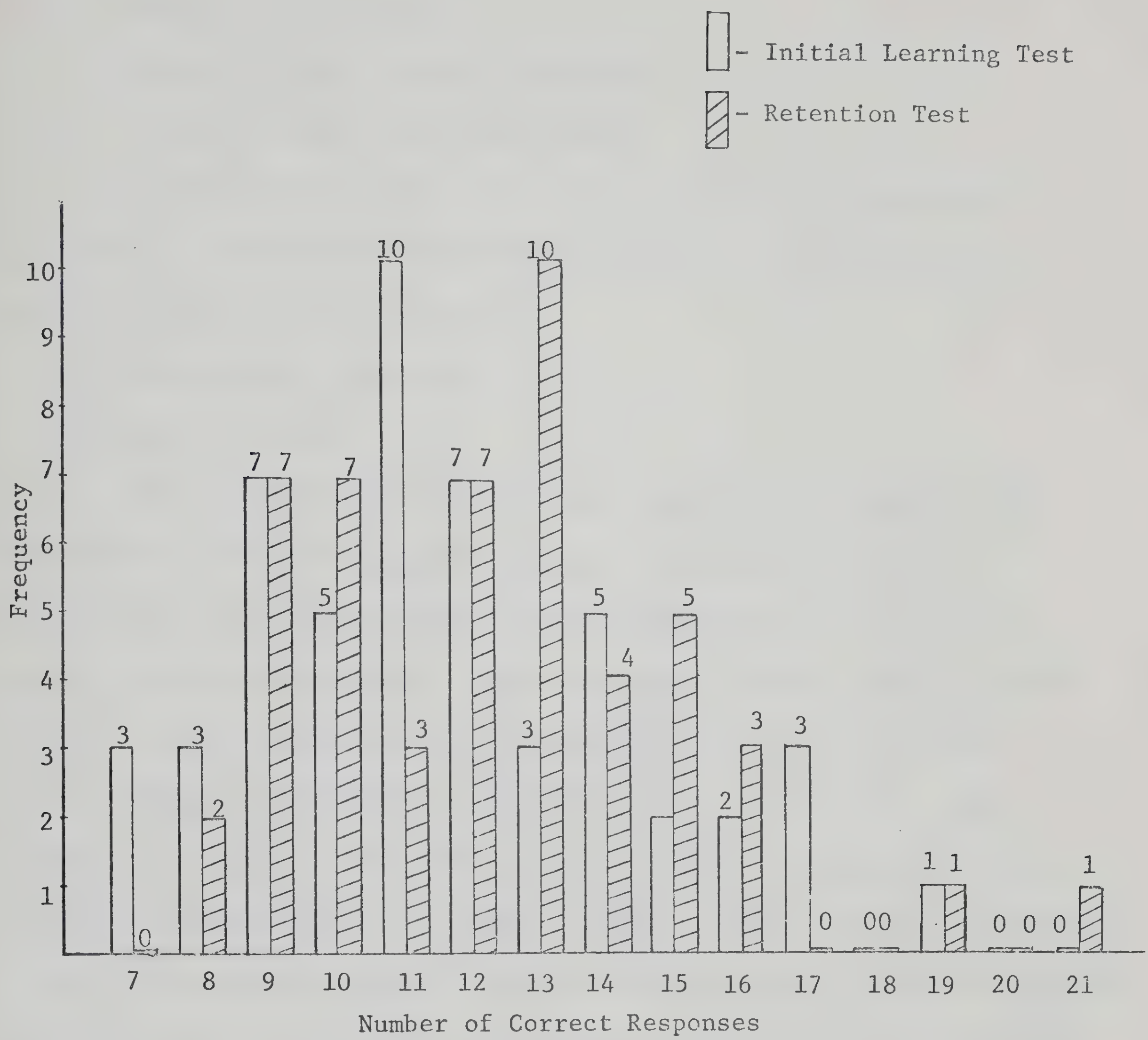

Figure 3

FREQUENCY DISTRIBUTION OF CORRECT RESPONSES FOR SD-SUBJECTS ON INITIAL LEARNING AND RETENTION TEST

The data associated with the variables and the tests described above were used to test the following null - hypotheses:

1. For subjects in the SD-group there exist no significant correlations between the initial learning test scores or retention test scores and the following variables:

a. intellectual ability

b. reading ability

c. socio-economic status

d. ability to make personal adjustment

e. ability to make social adjustment.

2. For the subjects in the SD-group there is no significant difference in mean initial learning test scores or retention scores of pupils who are grouped as being:

a. reflective or impulsive

b. male or female

Correlation coefficients (Pearson type) for the variables and the criteria were calculated and they are listed in Table VIII. No significant correlation existed between intelligence, socio-economic status, personal adjustment, social adjustment, reading ability and the scores for the Initial Learning and Retention Test. Therefore the corresponding null-hypotheses stated above were not rejected. There existed no significant differences in means between groups of: high or low intelligence, high or low socio-economic status, high or low ability to make personal adjustments, high or low ability to make social adjustments, or high or low reading ability on either the Initial Learning Test or the Retention Test. That is, for the self-directed group it was not possible to predict mathematics achievement as measured on the Initial Learning Test on the basis of the variables considered.

To test the hypotheses which deal with differences of groups formed on the basis of conceptual tempo and sex, _t_-ratios were calculated. These calculations are summarized in Table IX.

TABLE VIII

INTERCORRELATIONS*AMONG VARIABLES AND CRITERIA FOR SD-SUBJECTS

Variable	1	2	3	4	5	6	7	8	9	10	11
1. Retention Test	1.000										
2. Initial Learning Test	.642	1.000									
3. Intelligence	.222	.231	1.000								
4. Socio-economic Status	.197	.134	.175	1.000							
5. Total Adjustment	.005	-.117	.261	.016	1.000						
6. Personal Adjustment	.020	-.083	.197	.057	.937	1.000					
7. Social Adjustment	-.011	-.137	.292	.030	.931	.745	1.000				
8. MFF - Errors	-.124	-.235	-.045	.386	-.046	-.045	-.040	1.000			
9. MFF - Time	-.002	.199	-.280	-.113	-.173	-.044	-.285	-.449	1.000		
10. Reading	.136	.034	.561	-.115	.447	.345	.485	-.293	-.101	1.000	
11. Pretest	.331	.277	.154	.065	-.105	-.046	-.152	-.124	.185	.127	1.000

*Significant (α=.01) Correlation Coefficients are underlined.

TABLE IX

MEANS, STANDARD DEVIATIONS AND t -RATIOS OF INITIAL LEARNING AND RETENTION TEST SCORES FOR GROUPS FORMED ON THE BASIS OF SEX AND CONCEPTUAL TEMPO (SD)

Test	Group	N	Mean	Standard Deviation	t(calculated)*	
Initial Learning	Male	30	12.13	3.26	1.801	NS
	Female	21	10.71	1.82		
	Impulsive	17	11.35	3.56	.814	NS
	Reflective	18	12.16	2.23		
Retention	Male	30	12.50	3.19	.793	NS
	Female	20	11.85	2.18		
	Impulsive	16	12.56	3.69	.104	NS
	Reflective	18	12.67	1.97		

$^{*}t_{(.01,48)} = 2.686$

$^{*}t_{(.01,32)} = 2.450$

The results in Table IX indicate that there existed no significant differences in means on either the Initial Learning or Retention Test between boys and girls. Consequently, the corresponding null-hypotheses were not rejected. Although no significant differences in mean existed between boys and girls on the Initial Learning Test, the six highest scores, which ranged from 16 to 19, were obtained by six boys. The lowest marks on this test were scores of seven and eight. Of the six subjects who scored in this range, four were boys and two were girls.

Of the 17 subjects who were classified as impulsive, eight were boys and nine were girls. The reflective group was made up of eleven boys and seven girls. The differences in means on either of the two tests for the reflective and impulsive groups were not significant (Table IX) and, therefore, the corresponding null-hypotheses were not rejected.

Of the six subjects with the highest marks on the Initial Learning Test, three were classified as impulsive and two as reflective. Of the six subjects who had the lowest scores, four were impulsive and one was reflective.

It was previously stated that three and sometimes four of the SD-subjects did not stay with their group because they preferred to work by themselves. Three of these subjects were boys and one was a girl. A check of the scores these students obtained on the California Test of Personality showed that all four of them scored in the 20 percentile range or lower on the personal adjustment part of the test and the percentiles scores for the social adjustment subtest were 1, 5, 10, and 20. Only one of these subjects received a score lower than the mean for the group on the Initial Learning Test. Two of the subjects scored well above the mean, and the remaining subject obtained the highest score.

It is interesting to note that mathematics achievement, as measured on the Initial Learning Test, could not be predicted from the pretest scores for the subjects in this setting. The correlation coefficient of .277 between these two variables was not significant.

II. THE PARTIALLY TEACHER-DIRECTED TREATMENT (PTD)

The PTD-subjects wrote the pretest on the same day as the subjects for the other groups. There were 55 pupils in this group who wrote the test and their scores ranged from 0 to 13. The mean of the test was 6.38 and the standard deviation 2.37. The results of the test are summarized in Table X and Figure 4.

TABLE X

PRE-TEST RESULTS FOR PTD-SUBJECTS

Range	Mean	Standard Deviation
0 - 13	6.38	2.37

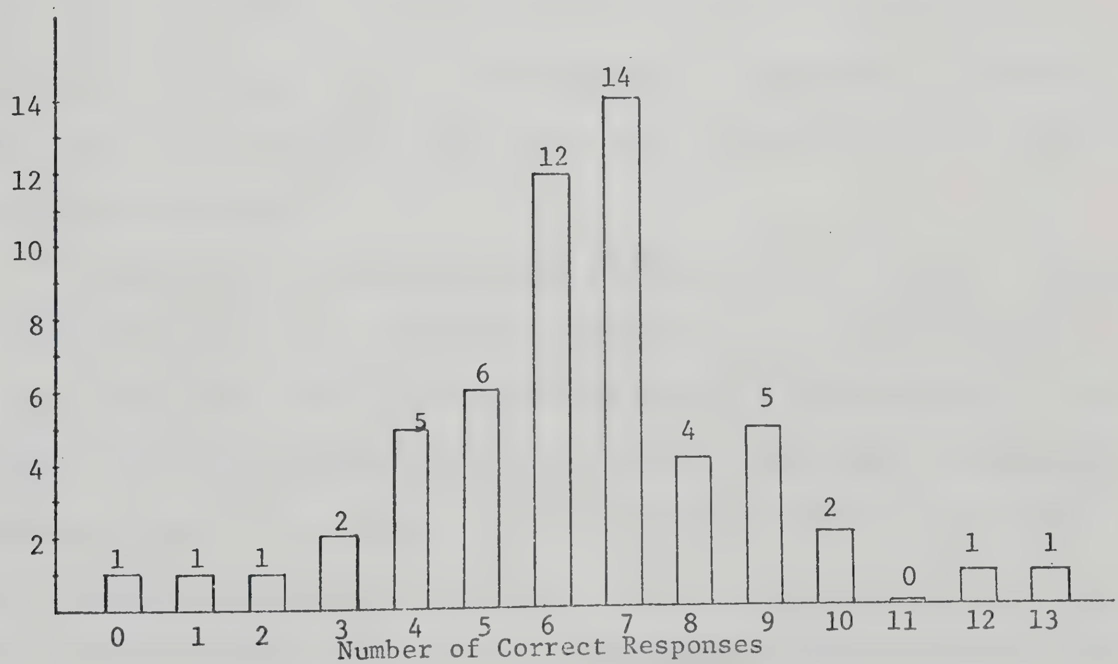

Figure 4

FREQUENCY DISTRIBUTION OF CORRECT RESPONSES ON PRETEST FOR PTD-SUBJECTS

While the study was in progress the PTD-pupils worked in an open-area in groups consisting of four pupils each. The students were assigned to these groups by their teachers who attempted to place at least one high achiever in each group. The majority of the groups consisted of boys and girls. Two boys and one girl preferred to work by themselves and did so for the duration of the study.

The setting for this group could be described as a tightly organized one. A definite sequence was followed from day to day and it consisted of checking the previous day's exercises, introducing the topics for the day, assigning the problems and assisting the groups that requested help. For the checking of the exercises, the teacher read out the answers. The two teachers took turns in introducing the topics. A typical introduction to a topic included the reading of a paragraph, explaining some of the terminology, solving a sample problem and assigning the problems for the period. The pupils were encouraged to work as a group while solving the problems. For the remaining part of each period the teachers moved about the area giving help to and making suggestions for the different groups that requested it.

The teachers insisted that the students work quietly and, consequently, the noise level was low. The pupils did exchange ideas but most of the communication that took place was between pupils in the same group. There existed very little interaction between groups. If a group had completed the daily tasks they proceeded to the next problem sheet or they solved some of the optional activities.

The intelligence, personality, and reading tests were administered to the group by the experimenter while the study was in progress. The results of these tests are given in Table XI.

TABLE XI

RANGES, MEANS AND STANDARD DEVIATIONS OF INTELLIGENCE - , PERSONALITY - AND READING TEST SCORES FOR PTD-SUBJECTS

Test	Range	Mean	Standard Deviation
Intelligence	75 - 134	109.64	13.46
Personality adjustment			
Total	56 - 133	95.34	19.62
Personal	21 - 67	45.55	10.86
Social	28 - 67	49.79	9.99
Reading	10 - 56	39.21	12.43

The occupations of the father, and in one case of the mother, were used to get a measure of socio-economic status for each subject. The range, mean and standard deviation of the Occupational Class Scale values (Blishen, 1968) are shown in Table XII.

TABLE XII

RANGE, MEAN AND STANDARD DEVIATION OF BLISHEN OCCUPATIONAL CLASS SCALE VALUES FOR PTD-SUBJECTS

Range	Mean	Standard Deviation
29.2 - 74.3	50.27	14.17

The MFF-Test was administered to each of the subjects individually by the experimenter. The time taken to give the first response was recorded with a stopwatch and a record was kept of each reply until the right answer was given. The average time taken for each question and the average number of errors per question were calculated for each pupil. The ranges, means and standard deviations for these two variables are presented in Table XIII.

TABLE XIII

RANGES, MEANS AND STANDARD DEVIATIONS OF
MFF-TEST RESULTS FOR PTD-SUBJECTS

	Range	Mean	Standard Deviation
Time	6.5 - 46.5	22.02	9.87
Errors	.17 - 2.33	.90	.50

A scatter diagram of average response time against the average number of errors was plotted to classify the subjects as impulsive or reflective. On the basis of this diagram it was decided to define an impulsive subject as a pupil whose average response time was less than 20 seconds and who averaged more than .83 errors per question. A subject who took more than 20 seconds for each question and whose average error rate was less than 1.00 was classified as a reflective individual. This method of division resulted in 17 subjects being classified as reflective and 15 subjects as impulsive. The results of the MFF - Test are summarized in the form of a scatter diagram in Figure 5.

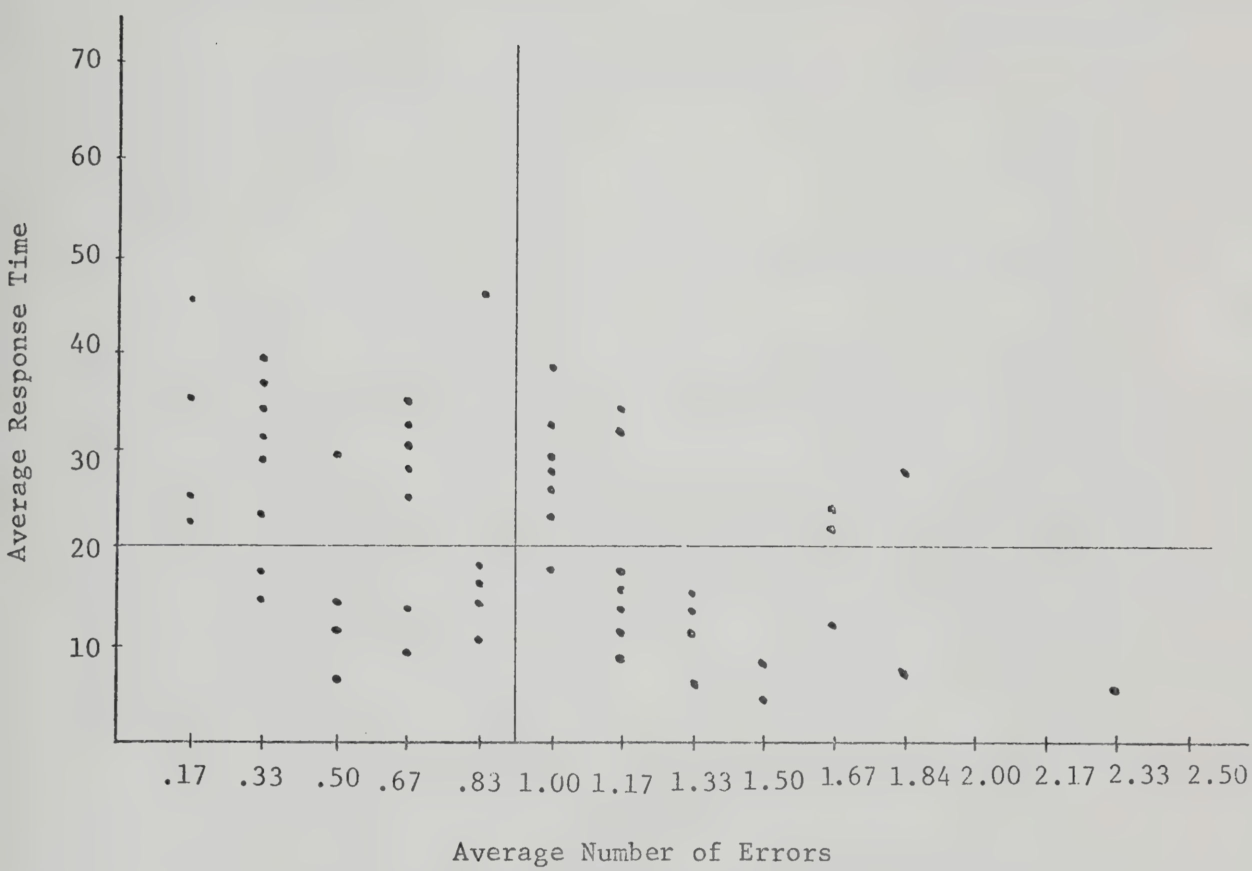

Figure 5

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN TIME AND ERRORS ON MFF-TEST FOR PTD-SUBJECTS

The Initial Learning Test which consisted of 30 multiple choice items was administered one day after the study was completed. The scores for this group ranged from 7 to 21 and the mean was 12.34. The results for the different parts of the test and for the whole test are listed in Table XIV. The frequency distribution of the correct responses is shown in Figure 6.

TABLE XIV

MEANS AND STANDARD DEVIATIONS OF INITIAL LEARNING
TEST SCORES FOR PTD-SUBJECTS

	Mean	Standard Deviation
Part A	6.21	2.24
Part B	3.58	1.62
Part C	2.55	1.20
Total	12.34	3.66

The results of the same test administered four weeks later as a measure of retention are presented in Table XV. For this test the scores ranged from 3 to 22 and the mean was 12.66. The distribution of the scores are illustrated in Figure 6.

TABLE XV

MEANS AND STANDARD DEVIATIONS OF RETENTION TEST
SCORES FOR PTD-SUBJECTS

	Mean	Standard Deviation
Part A	6.32	2.18
Part B	3.40	1.98
Part C	2.94	1.29
Total	12.66	4.31

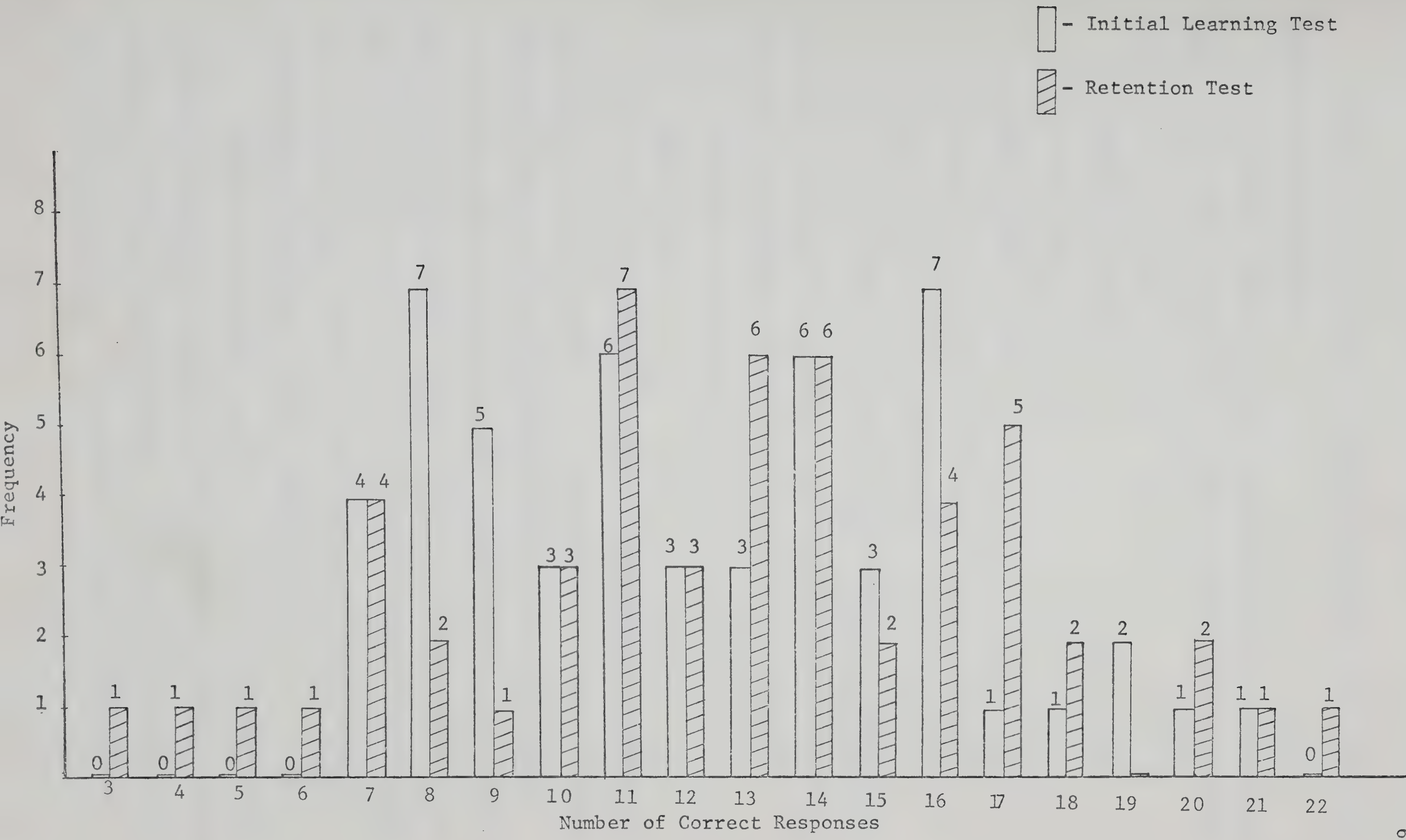

Figure 6

FREQUENCY DISTRIBUTION OF CORRECT RESPONSES FOR PTD-SUBJECTS
ON INITIAL LEARNING AND RETENTION TEST

The data presented on the previous pages were used to test the following null-hypotheses:

3. For the subjects in the PTD-group there exist no significant correlations between the initial learning test scores or retention test scores and the following variables:

a. intellectual ability

b. reading ability

c. socio-economic status

d. ability to make personal adjustment

e. ability to make social adjustment

4. For the subjects in the PTD-group there is no significant difference in mean initial learning test scores or retention scores of pupils who are grouped as being:

a. reflective or impulsive

b. male or female

Correlation coefficients (Pearson type) between the variables mentioned in the null-hypotheses were calculated and they are listed in Table XVI. Significant relationships existed between intelligence, personal adjustment, social adjustment, reading ability and the Initial Learning and Retention Test scores. Therefore the corresponding null-hypotheses stated above were rejected. Since the relationships between socio-economic status and the two test scores were not significant, the associated null-hypotheses were not rejected.

Of the variables that resulted in a significant relationship with the learning and retention test scores, only intelligence correlated significantly with the pre-test scores. The correlation coefficients for the pretest scores and the variables are shown in Table XVII.

TABLE XVI

INTERCORRELATIONS* AMONG VARIABLES AND CRITERIA FOR PTD-SUBJECTS

Variable	1	2	3	4	5	6	7	8	9	10	11
1. Retention Test	1.000										
2. Initial Learning Test	.740	1.000									
3. Intelligence	.764	.650	1.000								
4. Socio-economic Status	.291	.313	.378	1.000							
5. Total Adjustment	.627	.532	.585	.191	1.000						
6. Personal Adjustment	.543	.453	.475	.167	.928	1.000					
7. Social Adjustment	.641	.553	.633	.193	.918	.757	1.000				
8. MFF - Errors	-.472	-.297	-.560	-.190	-.534	-.513	-.491	1.000			
9. MFF - Time	.387	.273	.463	.104	.287	.226	.317	-.440	1.000		
10. Reading	.742	.623	.802	.237	.657	.542	.702	-.437	.358	1.000	
11. Pretest	.449	.411	.391	.172	.321	.312	.289	-.270	.086	.314	1.000

*Significant ($\alpha = .01$) Correlation coefficients are underlined.

The results obtained seem to indicate that for the group in this setting which was partially teacher-directed and where the pupils worked in groups such variables as personal adjustment, social adjustment, and reading ability are of importance in predicting the learning outcomes as measured on the Initial Learning Test.

TABLE XVII

INTERCORRELATIONS*AMONG VARIABLES AND PRE-TEST SCORES FOR PTD-SUBJECTS

	Intelligence	Socio-economic Status	Personal Adjustment	Social Adjustment	Reading
Pre-test	.<u>391</u>	.172	.312	.289	.314

*Significant (α = .01) Correlations are underlined

To test the differences in means between groups formed on the basis of conceptual tempo and sex, *t*-ratios were calculated. The results of these calculations are presented in Table XVIII.

The results of the calculation listed in Table XVIII show that there existed no significant differences between boys and girls on either the Initial Learning or Retention Test. Thus, the corresponding null-hypotheses were not rejected. Although no significant differences in means existed between boys and girls, the five highest scores on the Initial Learning Test, which ranged from 18 to 21, were obtained by boys. Four subjects received a score of seven on this test. This group consisted of two girls and two boys.

Fifteen subjects made up the impulsive group. Of these seven were boys and eight were girls. The reflective group consisted of eleven boys and six girls. The mean for the reflective subjects on the Initial Learning Test was higher than for the impulsive subjects, but this difference was not significant at the .01 level. Consequently, the corresponding null-hypothesis presented was not rejected. However, on the retention test the reflective students scored significantly higher than the impulsive students, and, therefore, the appropriate null-hypothesis was rejected.

On the pre-test the reflective subjects had a mean of 7.43 and the mean for the impulsive subjects was 5.74. The calculated $\underline{t}$-ratio value was 2.106. This difference in means was not significant, since $t(.01,30)= 2.457$.

Of the five subjects who scored highest on the Initial Learning Test, three were classified as reflective and one as impulsive. Of the four subjects with the lowest marks, two were impulsive subjects and one was reflective.

TABLE XVIII

MEANS, STANDARD DEVIATIONS AND t-RATIOS OF INITIAL LEARNING AND RETENTION TEST SCORES FOR GROUPS FORMED ON THE BASIS OF SEX AND CONCEPTUAL TEMPO (PTD)

Test	Group	N	Mean	Standard Deviation	t(calculated)*	
Initial Learning	Male	32	12.56	3.99	.538	NS
	Female	21	12.00	3.26		
	Impulsive	15	11.33	4.02	1.945	NS
	Reflective	17	14.00	3.72		
Retention	Male	32	13.13	5.12	.966	NS
	Female	21	11.95	2.65		
	Impulsive	15	10.87	3.05	3.910	S
	Reflective	17	15.82	2.98		

$*t_{(.01,51)} = 2.680$

$*t_{(.01,30)} = 2.457$

Three subjects, two boys and one girl, of the PTD-group left the groups they were assigned to and worked by themselves. Their percentile scores for the total and the subtests of the California Test of Personality ranged from five to thirty. One of these subjects was excluded from the study since he failed to complete any of the tasks or tests presented to him. The scores on the Initial Learning Test for the other two subjects were near the mean for the group.

A significant correlation coefficient of .411 between the pretest scores and the Initial Learning Test scores indicated that mathematics achievement could be predicted for the subjects in this setting on the basis of prior knowledge.

III. THE TEACHER - DIRECTED TREATMENT (TD)

The subjects for the SD- and PTD-treatments attended one school and the subjects selected for the TD-treatment came from a second school. All of the grade-five pupils from this latter school were used in the study and there were 38 subjects who wrote the pre-test. The scores for this test ranged from 0 to 14. The mean was 7.71 and the standard deviation was 2.91. The results of this test are summarized in Table XIX and Figure 7.

TABLE XIX

PRE-TEST RESULTS FOR TD-SUBJECTS

Range	Mean	Standard Deviation
0 - 14	7.71	2.91

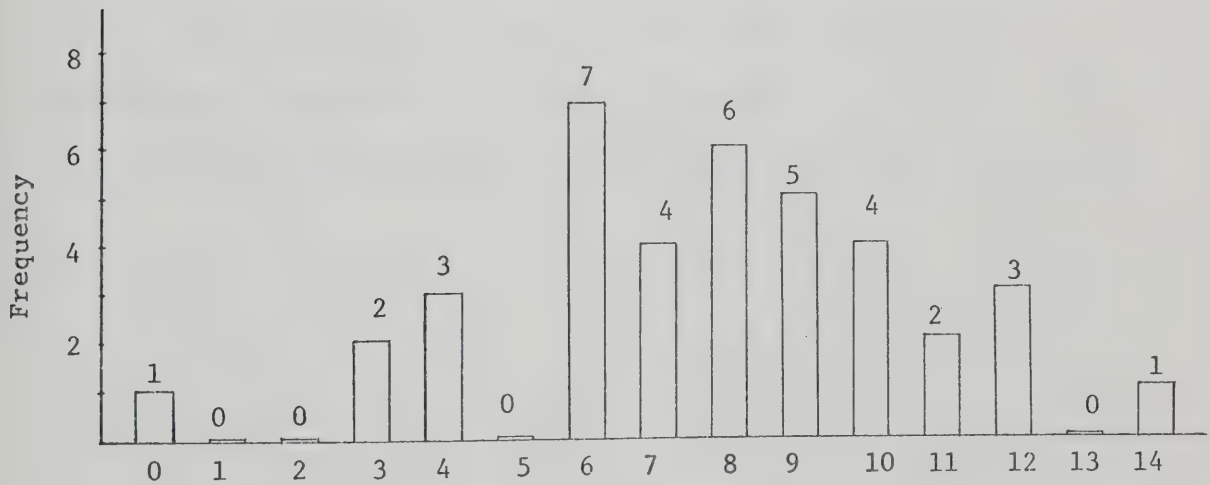

Figure 7

FREQUENCY DISTRIBUTION OF CORRECT RESPONSES ON PRE-TEST FOR TD-SUBJECTS

While the study was in progress the subjects of the TD-group met in one part of an open-area for their daily mathematics class. Their tables and chairs were arranged in rows and the pupils worked by themselves. For the duration of the study two teachers were available, but only one of them was in charge of the group at any one time.

A typical lesson consisted of an introduction or explanation, the assigning of problems and the checking of answers. Any questions that arose during the class period were answered on an individual basis and, frequently, explanatory diagrams were drawn on the blackboard for the pupil. Questions and corresponding explanations were a common occurrence and often four or five subjects would listen to an explanation given to a specific student. The pupils were asked to work by themselves, but, nevertheless, there existed frequent exchanges of ideas and answers between students who sat near one another. The teacher's favorite position appeared to be close to a blackboard and students would leave their desks to receive help which was often given with the aid of the blackboard.

The group tests were administered to the subjects by the experimenter during the first week of the study. Some of the results for the intelligence, personality and reading test are listed in Table XX.

TABLE XX

RANGES, MEANS AND STANDARD DEVIATIONS OF INTELLIGENCE-, PERSONALITY - AND READING TEST SCORES FOR TD-SUBJECTS

Test	Range	Mean	Standard Deviation
Intelligence	99 - 130	117.62	8.87
Personality Adjustment			
Total	67 - 131	106.05	15.80
Personal	26 - 65	50.84	9.24
Social	36 - 67	55.22	7.86
Reading	21 - 55	44.81	7.21

A measure of each subject's socio-economic status was obtained by matching the father's occupation with the occupations listed on Blishen's Class Scale. The range, mean and standard deviation for these values are given in Table XXI.

TABLE XXI

RANGE, MEAN AND STANDARD DEVIATION OF BLISHEN OCCUPATIONAL CLASS VALUES FOR TD-SUBJECTS

Range	Mean	Standard Deviation
29.3 - 76.0	62.42	13.16

The MFF - test was administered to each subject. The responses to the test were timed and recorded. The average response time for each question and the average number of errors for each question were calculated for each subject. The ranges, means and standard deviations for these results are summarized in Table XII.

TABLE XXII

RANGES, MEANS AND STANDARD DEVIATIONS OF MFF - TEST RESULTS FOR TD-SUBJECTS

	Range	Mean	Standard Deviation
Time	8.0 - 91.5	33.07	19.52
Errors	0.0 - 1.83	.70	.45

The average response time was plotted against the average number of errors per question for each subject and the scatter diagram (Figure 8) was used to classify the subjects as impulsive and reflective. An impulsive individual was defined as one whose average response time was less than 30 seconds and who averaged more than .50 errors per question. The reflective subject, therefore, took longer than 30 seconds for each question and he made less than .67 errors for each question. As a result of this division 14 subjects were classified as being impulsive and 12 subjects as reflective.

The MFF-test scores for the subjects in this group resulted in a classification scheme with respect to reflectiveness and impulsiveness, which differed when compared to the SD- and the PTD-groups. The cut-off points in terms of time and errors were different. However, this is no way effected the analysis since the groups for the different treatments were not compared.

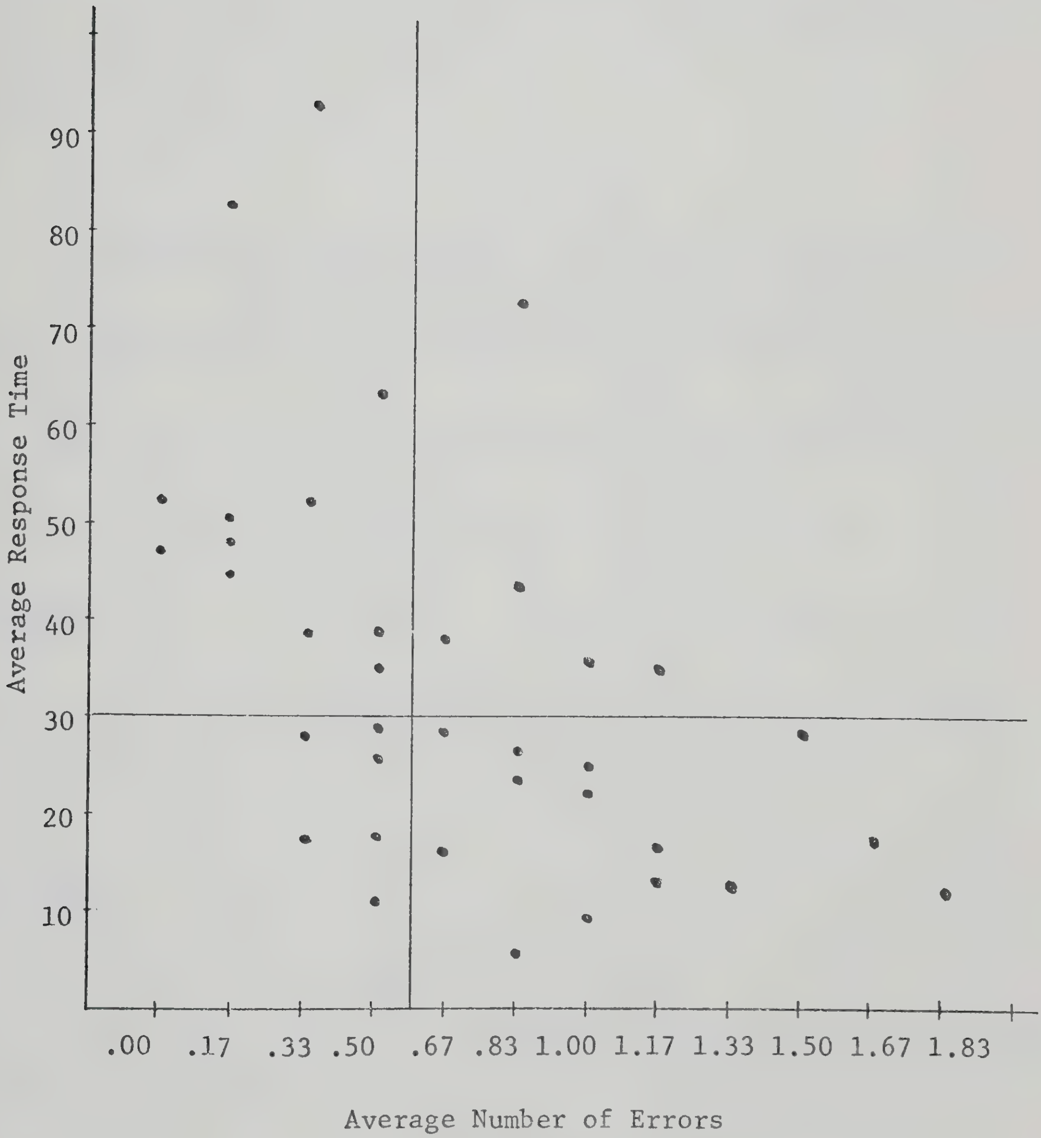

Figure 8

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN TIME AND ERRORS ON MFF-TEST FOR TD-SUBJECTS

The three-part 30 multiple choice item Initial Learning Test was administered to the subjects one day after the study was completed. The scores for this test ranged from 9 to 26 and the mean was 15.89. The mean for the Retention Test, which was administered four weeks later, was 17.28 and the scores ranged from 11 to 26. The mean and standard deviation for both the Initial Learning and Retention Test are presented in Table XXIII. The frequency distribution of the correct responses for both tests is shown in Figure 9.

TABLE XXIII

RANGES, MEANS AND STANDARD DEVIATIONS OF INITIAL LEARNING AND RETENTION TEST SCORES FOR TD-SUBJECTS

Test	Part	Mean	Standard Deviation
Initial Learning	Part A	7.73	1.87
	Part B	4.92	2.13
	Part C	3.19	1.05
	Total	15.89	3.66
Retention	Part A	8.31	1.53
	Part B	5.33	1.88
	Part C	3.61	1.02
	Total	17.28	3.21

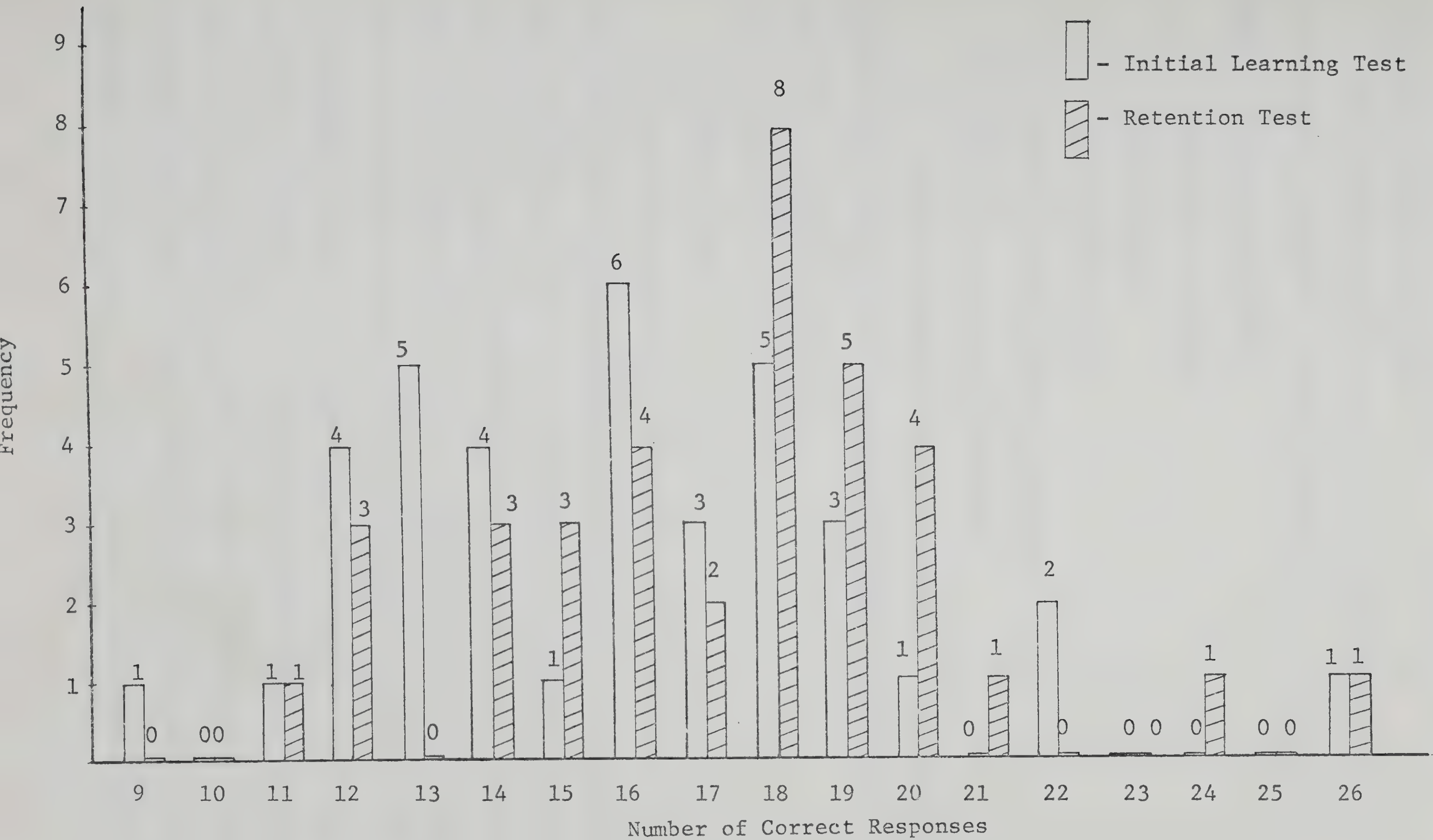

Figure 9

FREQUENCY DISTRIBUTION OF CORRECT RESPONSES FOR TD-SUBJECTS ON INITIAL LEARNING AND RETENTION TEST

The data on the variables outlined in the previous paragraphs and the results of the Initial Learning and Retention Test were used to test the following null-hypotheses:

5. For the subjects in the TD-group there exist no significant correlations between the initial learning test or retention test scores and the following variables:

a. intellectual ability

b. reading ability

c. socio-economic status

d. ability to make personal adjustment

e. ability to make social adjustment

6. For the subjects in the TD-group there is no significant difference in mean initial learning test scores or retention scores of pupils who are grouped as being:

a. reflective or impulsive

b. male or female

Correlation coefficients (Pearson type) between the variables and the Initial Learning and Retention Test scores were calculated and these intercorrelations are listed in Table XXIV. Correlation coefficients of .466 and .474 between intelligence and the Retention and Initial Learning Test scores, respectively, indicate that the pupils of high intelligence scored significantly higher on these tests than the pupils of low intelligence. Thus the corresponding null-hypotheses were rejected. A correlation coefficient of .378 between intelligence and the pretest scores was not significant at the .01 level. No significant relationships existed between socio-economic status, personal adjustment, social adjustment and the scores for the two tests. Therefore, the corresponding

TABLE XXIV

INTERCORRELATIONS* AMONG VARIABLES AND CRITERIA FOR TD-SUBJECTS

Variable	1	2	3	4	5	6	7	8	9	10	11
1. Retention Test	1.000										
2. Initial Learning Test	<u>.644</u>	1.000									
3. Intelligence	<u>.466</u>	<u>.474</u>	1.000								
4. Socio-economic Status	.042	.285	.050	1.000							
5. Total Adjustment	-.020	-.106	.071	-.004	1.000						
6. Personal Adjustment	-.078	-.208	-.005	.066	<u>.935</u>	1.000					
7. Social Adjustment	.050	.031	.149	-.085	<u>.910</u>	<u>.704</u>	1.000				
8. MFF - Errors	-.140	-.125	-.134	.129	-.302	-.209	-.362	1.000			
9. MFF - Time	-.019	-.027	-.006	-.161	.305	.182	<u>.393</u>	<u>-.533</u>	1.000		
10. Reading	<u>.424</u>	<u>.410</u>	<u>.625</u>	.045	.111	.008	.214	-.129	-.216	1.000	
11. Pretest	<u>.449</u>	<u>.479</u>	.378	.135	.196	.086	.292	-.249	.071	<u>.418</u>	1.000

*Significant (α = .01) Correlation Coefficients are underlined.

null- hypotheses were not rejected. The relationships between reading and the Initial Learning and Retention Test scores was significant and, consequently, the corresponding null-hypotheses were rejected. A correlation coefficient of .418 between reading and the pretest scores was significant.

To test the null-hypotheses concerned with mean differences for conceptual tempo and sex groups, t - ratios were calculated. The results of the calculations which are presented in Table XXV indicate that there existed no significant differences between means for boys and girls on either the Initial Learning or Retention Test. Therefore, the

TABLE XXV

MEANS, STANDARD DEVIATIONS AND t-RATIOS OF INITIAL LEARNING AND RETENTION TEST SCORES FOR GROUPS FORMED ON THE BASIS OF SEX AND CONCEPTUAL TEMPO (TD)

Test	Group	N	Mean	Standard Deviation	t*(calculated)
Initial Learning	Male	16	15.81	4.59	.036 NS
	Female	21	15.86	2.86	
	Impulsive	14	16.14	3.35	.047 NS
	Reflective	12	16.08	3.06	
Retention	Male	16	17.25	4.04	.000 NS
	Female	20	17.25	2.47	
	Impulsive	14	17.50	3.29	.041 NS
	Reflective	11	17.45	1.81	

$^{*}t_{(.01,34)} = 2.732$ $\qquad$ $t_{(.01,24)} = 2.494$

associated null-hypotheses were not rejected. The four highest marks on the Initial Learning Test ranged from 20 to 26. Three boys and one girl scored within this range. Three boys and three girls made up the group with the six lowest marks which ranged from 9 to 12.

The method employed to group the subjects according to conceptual tempo resulted in twelve subjects being classified as reflective and fourteen as impulsive. Two of the reflective students were boys and ten girls. The impulsive group consisted of nine boys and five girls. No significant differences in means on the Initial Learning and the Retention Test existed between the reflective and impulsive group. Consequently, the corresponding null-hypotheses were not rejected.

The group of four subjects who scored highest on the Initial Learning Test had two reflective subjects and one impulsive subject as its members. Of the six subjects who received the lowest scores, two were classified as impulsive. There were no reflective subjects in this group.

Mathematics achievement, as measured on the Initial Learning Test, could be predicted from the pretest scores for the subjects in this setting. The correlation coefficient of .479 between these two variables was significant.

IV. THE THREE TREATMENTS

The main point of this study has to do with the examination of the treatment groups, separately, in relation to a number of variables. As the means on the tests used to measure these variables indicate, the characteristics of the groups with respect to these variables differed (Tables III, IV, XI, XII, XX and XXI). Keeping these differences in mind there might be some value in making a crude comparison.

When differences in means on the Initial Learning Test and the Retention Test for the three treatment groups are compared, no important differences exist between the SD- and the PTD-groups. However, the mean for the subjects in the TD-group was higher than the means for the subjects in the other two groups.

V. SUMMARY OF THE RESULTS

In the previous three sections of this chapter the data and the results for the three treatments and the corresponding hypotheses were presented. In this section the purpose of the study and the main findings are summarized.

The main purpose of the study was to investigate the relevance in different settings of mathematics teaching of such pupil characteristics as intellectual ability, ability to make personal adjustment, ability to make social adjustment, reading ability, socio-economic status, conceptual tempo and sex. To evaluate the importance of these variables three distinct settings were studied - one self-directed, another partially teacher-directed and the third teacher-directed. Initial learning and retention tests were administered to the subjects in each of these settings. The learning test was completed immediately following the study and the retention test four weeks later. The results of these tests were used to study the relationship between learning outcomes and the variables listed above. All of the subjects also wrote a pretest that consisted of questions which dealt with some of the concepts presented during the study.

For the subjects in the self-directed setting none of the factors: intelligence, ability to make personal adjustment, ability to make social adjustment, reading ability and socio-economic status correlated significantly with either the Initial Learning or Retention Test scores. There existed no significant differences in means on these two tests between boys and girls or between subjects who were classified as reflective or impulsive.

For the subjects in the partially teacher-directed setting there existed significant correlations between intelligence, personal adjustment, social adjustment, reading ability and the Initial Learning and Retention Test scores. Of these variables, only intelligence showed a significant relationship with the pretest scores. No significant correlation existed between socio-economic status and the two test scores. The subjects who were classified as reflective obtained a higher mean on the Initial Learning and Retention Test than the impulsive subjects, but only the Retention Test means differed significantly. The means on the pretest for these two groups did not differ significantly. The Initial Learning and Retention Test means for boys and girls did not differ significantly.

For the subjects in the teacher-directed setting the correlation coefficients for intelligence and the Initial Learning and Retention Test scores were significant. The correlation between intelligence and the pretest scores was not significant. No significant relationship existed between socio-economic status, personal adjustment, social adjustment and either the Initial Learning or the Retention Test scores. The correlation between reading ability and the two test scores was significant. However, this significant relationship also existed for the pretest scores. There existed no significant differences in means between impulsive and

reflective subjects or between boys or girls on either the Initial Learning or the Retention Test.

There existed no significant differences between boys and girls in any one of the three treatment groups. As was the case for the other variables, no significant relationship existed between conceptual tempo and the learning outcomes for the subjects in the self-directed group. In the partially teacher-directed group the subjects classified as reflective had a higher mean for the pretest, the Initial Learning Test and the Retention Test than the impulsive subjects, but only the means on the Retention Test differed significantly. In the teacher-directed group, the majority of the subjects classified as impulsive were boys, and most of the reflective subjects were girls. No difference in means existed between these two groups.

The amount of prior knowledge, as indicated by the pretest scores, could be used to predict the achievement results for the subjects in the partially teacher and the teacher-directed settings. No significant correlation existed between pretest scores and Initial Learning Test scores for the subjects in the self-directed setting, however.

CHAPTER V

SUMMARY, DISCUSSION AND IMPLICATIONS, RECOMMENDATIONS

I. SUMMARY AND CONCLUSIONS

There exist today at least two important suggestions for changes in the elementary mathematics classroom. One change that is suggested is the departure from teacher-dominated group instruction. A second suggestion for change implies that there should be less emphasis on the traditional study of operations and instead various aspects or categories of mathematics should be included in the program for elementary pupils. These two factors contributed to the development of this study.

It was the purpose of this study to investigate the relationship between such pupil characteristics as intellectual ability, the ability to make personal adjustment, the ability to make social adjustment, reading ability, socio-economic status, conceptual tempo (reflectiveness, impulsiveness) and sex and mathematics learning in three distinct settings. For each of the three settings the roles of the pupils and teachers differed.

For one of the settings the pupils worked in groups and they solved problems and participated in activities without receiving any formal instruction from the teachers. The pupils worked by themselves and the teachers' main task was to be on hand to answer questions or make suggestions. Answers for previous tasks were written on the blackboard, but checking was voluntary. This setting was labelled Self-Directed (SD).

For the second, or Partially-Teacher-Directed (PTD) setting, a few additional tasks were assigned to the teachers. The topics were formally introduced to the subjects in this group. After a brief formal introduction which usually included the reading of the answers from the previous day's work, explaining the new tasks for the day, giving some

examples and assigning the tasks for the day, the pupils would work in groups just as the SD-subjects did. The teachers were available to assist the groups.

A third, or more formal setting, was mainly Teacher-Directed (TD). The teacher supervised the checking of the work and explained the new topics to the pupils. The pupils worked by themselves, and assistance was given to them by the teacher on an individual basis.

An activity booklet (Appendix C) was prepared and it included exercises dealing with such topics as decimal fractions, number pairs, co-ordinates, perimeter, area, numbers in an array, contour lines, scale, networks, regions, multiplication, and probability. During a pilot study these topics were presented to a group of grade five students to make the items suitable for this grade level. The activities for the SD-and PTD-settings were the same and only a few minor revisions were made for the TD-group. These revisions included the elimination of activities or questions which implied working with a partner or as a member of a group and a part of the section on decimal fractions was rewritten (Appendix D). An activity booklet was prepared for each of the subjects in the study.

It was planned to use two classes for each of the settings described above. To control for the teacher variable it was decided to use open-area schools. In this way two class groups could be combined and the teachers could work together and exchange roles whenever the method of presentation called for a formal introduction of a topic.

A request was made to the Edmonton Public School Board office for six grade five classes from open-area schools. Two schools were included in the study. One hundred and eleven students representing four grade five classes attended one school and 38 students from one and one-half classes attended the other.

Two classes from the larger school, or 56 subjects, were assigned to the self-directed treatment. These subjects were chosen for this setting since they had had some experience with the activity approach to mathematics learning. Puzzles, games and other group activities were part of their program. The two remaining classes in this school, or 55 pupils, were assigned to the partially teacher-directed group. The 38 students in the second school made up the teacher-directed group. The students who missed five or more days during the study, which lasted about four weeks, were excluded. By the time the study was completed there were 51 subjects in the SD-group, 53 in the PTD-group and 37 in the TD-group. From these two more subjects, one from the SD-group and one from the TD-group, were absent when the Retention Test was given.

The school program for the two treatment groups in the same school was such that there existed very little communication or interaction between them. Their home-areas or home-rooms were located in different parts of the school and their daily schedules were different. When one group worked or studied in the open-area, the other group occupied classrooms in another part of the building.

To gather the necessary data several instruments were employed. While the study was in progress the following group tests were administered to the subjects: California Short-Form Test of Mental Maturity, California Test of Personality and one section of the Stanford Achievement Test - Reading. The Matching Familiar Figures Test was administered individually to each of the subjects. To determine a measure of socio-economic status the father's occupation was obtained from the school records and this was matched against the occupations listed on Blishen's revised Occupational Class Scale. All of the subjects wrote a pretest

(Appendix E) consisting of twenty multiple choice questions which were based on most of the topics presented during the study. A criterion test (Appendix B) which consisted of thirty multiple choice items was constructed and it was administered to the subjects one day after the study was completed and again four weeks later to obtain a measure of initial learning and retention respectively.

With the help of computer programs supplied by the Division of Educational Research Services the constructed tests were analyzed, descriptive data for each of the instruments used in the study were obtained, and intercorrelations between the variables were calculated. Group comparisons within treatments were made with the aid of a $\underline{t}$ - test function for IBM/67 that uses Iverson's APL-notation. The hypotheses were tested at the .01 level of significance.

For the subjects in the SD-group no significant relationships existed between intellectual ability, socio-economic status, ability to make personal adjustment, ability to make social adjustment and reading ability and the scores on either the Initial Learning Test or the Retention Test. There existed no significant differences for the means on the criterion tests between impulsive and reflective subjects and between boys and girls.

For the PTD-group intellectual ability, the ability to make personal adjustment, the ability to make social adjustment and reading ability correlated significantly with the scores on the Initial Learning Test and the Retention Test. Of these variables only intellectual ability resulted in a significant correlation with the pretest scores. The PTD-subjects classified as reflective had a higher mean for the pretest, the Initial Learning Test and the Retention Test than the impulsive subjects, but

only the means for the Retention Test differed significantly. There existed no significant difference between the means for boys and girls on either the Initial Learning or Retention Test.

The correlation coefficients for intellectual ability, reading and the Initial Learning and Retention Test scores were significant for the subjects in the TD-group. However, the correlation coefficient for reading ability and the pretest scores was also significant. The correlations between socio-economic status, ability to make personal adjustment, ability to make social adjustment and the Initial Learning or Retention Test scores were not significant. There existed no significant differences between impulsive and reflective subjects and boys and girls on either the Initial Learning or the Retention Test.

On the basis of the results presented the following conclusions may be drawn:

(1) The non-existent relationship between the variables and the learning outcomes for the subjects in the self-directed group suggests that changes in role or leadership style of the teacher results in different behavior reactions from the pupils. Even such factor as intellectual ability may fail to show up as reliable predictors of the learning outcomes in such a setting.

(2) When a partially teacher-directed setting is used as part of an instructional program, some types of learners stand to gain more from this procedure than others. In terms of mathematics achievement it would appear that students who make personal and social adjustment readily and are of high reading ability receive the greatest benefit from such a treatment.

(3) In a teacher-directed setting the most important of the variables studied as a predictor of mathematics achievement is intelligence of the child.

(4) Different levels of verbal interaction between pupils and teachers, and pupils and pupils, do not appear to produce differences in mathematics achievement between boys and girls.

(5) A setting which is self-directed will not be disadvantageous for an impulsive child. He will learn, for the first few weeks at least, as well as a reflective child.

In a partially teacher-directed setting reflective children tend to obtain higher scores on an achievement test than impulsive children. Reflective children also retain more of the material when it is presented in such a setting.

(6) When independent group work becomes an important part of an instructional program, most students appear to adjust easily to such a setting. They appear to work eagerly and productively at the tasks presented to them. However, an achievement test administered after a few weeks of such a program will not be a good indicator of the productivity that seems apparent.

(7) In terms of learning outcomes, formal instruction given to a group as a whole at the beginning of a class session appears to be of little value.

(8) The ability to predict mathematics achievement on the basis of prior knowledge appears to depend on the setting in which mathematics is taught.

II. DISCUSSION AND IMPLICATIONS

For this study an attempt was made to simulate three different settings or three different methods of presenting topics in mathematics to three groups of grade-five students. The teachers' roles and the role of the pupils in each of the treatments differed.

Although three distinct settings were used it is probably wrong to assume that an elementary school teacher would just use one of these methods in his daily work with a group of students. As a teacher changes from one subject area to another, or even from one topic to another, he would likely use all three of the methods or even a combination of these. Even if there exists in the elementary schools today a trend toward instruction which is more self-directed for the pupils, the majority of the teaching, especially in mathematics, is still done in settings which are mainly teacher-directed.

Results from various studies are available which have investigated the relationship between such variables as intellectual ability, socio-economic status, reading ability and mathematics achievement in teacher-directed settings. Generalization from these results in terms of these variables would indicate that there exists a positive relationship between them and mathematics learning, but do these reationships change as the setting is changed or as the setting becomes more self-directed? What is the relationship between personality characteristics and mathematics learning for different settings? What are the effects of reflectiveness and impulsiveness as the method becomes more self-directed? Will there be any differences in performance between boys and girls?

This study was carried out to find the answers to the questions listed above. However, the discussion which follows must be interpreted while keeping the limitations of the study in mind. The reported results are based on a four week study, the treatments were assigned to the three groups, the three groups differed with respect to most of the variables considered here, and the measured learning outcomes are based on the scores of one test which was constructed for this purpose.

In the self-directed setting, the pupils had to extract the information that was needed to complete the activities from the written pages. In this setting the formal interaction between students and teachers was limited. Some groups of pupils worked two or even three class periods without talking to the teachers who were in charge. Some of the subjects worked on the activities, completed a set of exercises and continued onto the next set without doubting the correctness of their approach or work, and the teacher's help or assistance was not requested. Other groups of subjects received all the feedback they needed from the comparison of their responses with the answers of a group nearby and they based the correctness of their work on the degree of agreement of responses that existed between the two groups.

The results for the SD-group seem to indicate that the relationship between the variables included in this study and the learning outcomes as measured on the Initial Learning Test is interrelated with the role of the teacher. Usually a positive relationship exists between intelligence and mathematics achievement. Similar relationships have been found between socio-economic status and mathematics achievement and between reading ability and mathematics achievement. None of these relationships existed for the results collected from this group. It seems that when the

teacher's authority and direct influence is removed from the tasks to be performed, students who usually experience success fail to do so.

Since the students worked and interacted with each other throughout the study it was assumed that measures of personal and social adjustment might be related to mathematics achievement in such a setting. However, no such relationships existed. The results of the personality test could have been used to predict whether some of the subjects were able to work in or as part of a group or not, but this prediction in no way reflected the way these subjects performed on the achievement test.

The question, 'What kind of learner is best suited for this kind of an approach?' cannot be answered for the subjects in this sample since none of the variables included in this study were related to the measured learning outcomes. It can be suggested that when students are introduced to and work in a setting which is self-directed, for the first few weeks at least, variables or pupil characteristics other than those selected are related to success. If this suggestion is correct, an awareness of this fact by the classroom teacher could be of importance since it may prevent him from drawing wrong conclusions. He should keep in mind that when he changes from one teaching strategy to another his usually reliable method of predicting success for some of his students may no longer be valid. The study produced two examples. When the students were assigned to groups by the teachers an attempt was made to place a high achiever into each group. To the teachers high achiever and high intelligence were synonymous. However, the assumed relationship did not hold for the subjects in this setting and the results the teachers had had in mind did not materialize. Implicitly, the students who worked by themselves were thought of as subjects who had little chance of completing their tasks as

successfully as the remaining subjects. However, the results proved otherwise.

It is interesting to note that no important differences existed between the standard deviations of the pretest for the three treatment groups (Tables II, X, and XIX). However, the results in Tables VI, XIV and XXIII indicate that the standard deviations for the Initial Learning and Retention Test were smallest for the SD-group. This seems to suggest that a self-directed setting may perhaps produce a uniformity of performance when mathematics achievement is considered.

The teacher's role for the partially teacher-directed setting was more defined than for the setting described above. The subjects belonging to this group were exposed to two distinct parts for most of their mathematics sessions. During the first part, topics were formally introduced and a definite sequence of tasks was prescribed to the subjects by the teacher. During the second part, the subjects were required to work quietly in their groups. As a result there existed little interaction between the groups. To get a teacher's attention the subjects would raise their hands and then wait until some assistance was given. Most of the groups waited for some feed-back from one of the teachers before they continued with their work.

When compared to the self-directed group, the teacher in the partially teacher-directed setting had a very direct influence on the subjects and the tasks that were to be performed by them. This teacher influence appears to be directly related to the abilities to make personal and social adjustment on the behalf of the students since the learning outcomes for the subjects in this group could be predicted on the basis of these two variables. Intellectual and reading ability could also be used to predict

the learning outcomes for the subjects in this group.

If it is true that students of high intellectual ability, high ability to make personal adjustment, high ability to make social adjustment and high reading ability receive the greatest benefit from teacher-directions, then the question about teacher-effectiveness must be raised. It has been said that this kind of student will 'survive' any method and learn no matter what method is used. How then could or should a teacher revise his method to be of maximum benefit for the remaining subjects? Could the class be divided into groups and one group work in a partially teacher-directed setting or even self-directed setting, while the other group receives the benefit of his guidance? If the abilities to make personal and social adjustment are related to success in a partially teacher-directed setting how would it be possible to accommodate subjects who lack feelings of personal and social security? The positive relationship between reading ability and the learning outcomes leads to further questions. When activities that involve the extracting of information by doing a certain amount of reading are presented should the subjects who have reading difficulties be grouped together and special attention be given to them? Or should an attempt be made to place only one of these subjects into a group so that this student can benefit from the reading skills the other members of the group possess? Finally, the intercorrelations listed in Table XVI (p. 62) show that there exists a significant relationship between intellectual ability and the variables labelled personal adjustment, social adjustment and reading. Do these variables represent different abilities or pupil characteristics or are they only different labels for the same thing?

The SD-group and the PTD-group came from the same school. The different classes in these groups were made up at the beginning of the school year and, according to the teachers, there was no reason to suspect that there existed any difference between the treatment groups with respect to the variables considered in this study. The means for these variables listed in Tables III, IV, XI and XII in chapter three give some indication as to the make-up of the two groups. The personality test means for the two groups were about the same. However, for the remaining variables, intelligence, reading and socio-economic status, the means for the SD-group were lower. The greatest difference in means existed for socio-economic status. The means for the Initial Learning and Retention Test, which are shown in Tables VI, VII, XIV and XV, suggest that at the end of the study no important differences existed between the SD-group and the PTD-group. This result seems to imply that the teacher responsibilities and tasks characteristic of the PTD-group did not constitute important factors which contributed to the learning outcomes of the group. The most important task the teachers in this group had was the formal introduction of a topic to the group as a whole. If it is true that this procedure does not contribute to the learning outcomes of a group, then which of the daily tasks a teacher performs in a classroom does? Probably the assistance given to an individual is the most valuable contribution a teacher can make. If students are working in groups, then it could be that individualization of instruction within groups or the direct assistance given to an individual in the group is more important than addressing a group as a whole.

The setting for the third group was purely teacher-directed. The teacher in charge of this group outlined the tasks that were to be performed by the individuals in this group and there existed little interaction between the subjects. All of the questions raised were directed toward the teacher and the replies given usually concerned only one individual member of the group. At times two or three subjects who sat close together would participate in the same conversation with the teacher. Sometimes two subjects would get together and exchange ideas or talk about something they had done or completed.

The TD-subjects were selected from a different school and this school was located in a different area of the city. On the basis of the means for the variables which were considered in this study, it can be concluded that the subjects in this group are representative of a population that is quite different than the population for the SD-and the PTD-subjects. The means of all the variables for the TD-subjects (Tables XX and XXI) were higher than the means of these variables for the other two groups. This was also the case for the means on the Initial Learning and Retention Test.

The positive relationship that existed between intellectual ability and the Initial Learning and Retention Test scores for the subjects in this treatment is in agreement with the results of many other studies which have reported findings on mathematics achievement and intelligence in teacher-directed settings. Since the subjects who had the highest IQ tended to get the highest scores on the Initial Learning Test, at least two important questions could be raised. How could a teacher change his approach or method in order to be just as effective for the students of lower intelligence? Would the students of higher

intelligence do just as well in a setting that is less teacher-directed? If the latter is true, then the teacher could spend more time with the pupils who would benefit more from his added attention.

Since this group was very different from the SD-and PTD-group with respect to the variables considered, it would be interesting to know how such a group would fare in a self-directed or partially teacher-directed setting and what the relationship between the pupil characteristics and the learning outcomes would be.

III. COMMENTS AND SUGGESTIONS FOR FURTHER RESEARCH

As a result of the findings reported in this study and the observations carried out during the study many questions arise. Some of them have been stated in the preceding section, others are presented below. These questions are about the learners who took part in the study, the content that was presented to the subjects, and the teachers who participated.

The subjects in the self-directed and the partially teacher-directed treatments worked in groups. Some of these subjects freely exchanged ideas with each other and seemed to work well together while solving the tasks presented to them. Others contributed little to the group and were only interested in keeping up. In doing so, they copied what the other members of the group had recorded. A few individuals preferred to work by themselves and these pupils generally received low scores on the California Test of Personality, especially on the subtest which dealt with social adjustment. However, on the basis of the variables included in the study the subjects who helped in creating a favorable group

setting could not be identified. What specific characteristics do these pupils possess? How do they differ from the subjects who contribute little to the group?

Most of the questions that were posed to the teachers by the students were interpretive and they dealt with specific details such as "If there is not a blank, should we answer the question?" and "Is it alright if we do this in such a way?" It appeared that the groups that were busy and working well together did not ask these kinds of questions, and if they did they decided on a course of action without consulting the teachers. Yet, they appeared to ask each other questions. However, anytime an adult approached them, the process of fluent interaction seemed to be somewhat interrupted and unnatural. What kind of questions were asked in such a group? What kind of questions or exchanges of ideas did take place to make a group function as well? How did these subjects interact? How does the pattern of interaction differ from that of a group that does not seem to make any progress or function smoothly?

The Matching Familiar Figures Test makes possible the classification of students as impulsive or reflective. During the administration of the test few or no differences existed between students who reacted quickly to the stimuli presented to them. However, the pupils who took their time in solving the problems presented to them used different methods. Some of them glanced quickly over the pictures to find the correct one and repeated this process several times. Others looked back and forth, eliminated one picture at a time, and classified it as correct or incorrect. A few subjects solved the problem of finding the correct picture by looking at the stimulus only once. Some of the pupils did not identify the correct picture until they had found some-

thing wrong with the remaining pictures in the set, whereas others did not continue once they had identified a picture as the correct one. Why do these differences in approaches exist? Where do they originate? Are these differences related to perception or memory? Do these differences result in different ways or methods of approaching a problem that is presented in the mathematics classroom? How do impulsive and reflective students differ in their daily classroom behavior? How do they differ in their behavior as part of a group?

Various comments made by the subjects of the teacher-directed group indicated that many of these subjects enjoyed doing the activities and exercises which were presented to them during the study. Some of them 'liked' them because they were different than the textbook, others stated that they 'liked' talking about different things (topics). It is true that in the elementary school today many teachers teach mathematics even though it is not their major field. Some of them teach it not because they want to but because they have to. For these teachers the textbook becomes the mathematics course and their students are taken through the book almost page by page. Listening to these children raised the question whether the textbook format itself might not have a direct relationship on attitudes towards mathematics. Could it be that following an almost identical format of mathematics learning year after year creates a negative attitude toward the subject? Does the attitude toward mathematics change when students use a number of textbooks? Does the attitude toward mathematics differ when students do not use a textbook at all?

Since a measure of attitudes was not included in this study, some obvious questions should be asked. What effect, if any, does a self-directed or partially teacher-directed program have on the attitudes of the subjects toward mathematics? Does the teacher or the type of material have a more direct influence on the attitudes toward mathematics? Can attitudes in students be changed by providing different settings?

At times the subjects in the self-directed and the partially teacher-directed groups were faced with questions that were intended to create a discussion. Some of the questions involved a comparison of answers and often these questions were included with the hope that they might result in a lively discussion or even an argument. However, when various individuals came across such a situation, or when they were faced with a question that did not have a blank to be filled in after it, they thought an error had occurred. For these children, questions that resulted in that kind of interaction were not for the mathematics classroom. What part of our mathematics teaching or what part of our mathematics program results in this kind of rigidity? How would the materials or the methods have to be changed to overcome this?

Throughout any one given day an elementary school teacher uses many different methods of teaching. However, these different methods seem to be associated with different subject areas. For example, many consider that experiments are for science, research and collecting data are for social studies, creativity is a part of the art program and individual tasks and performances are part of the new physical education program. Various subjects make use of a combination of these methods. When it comes to mathematics, teachers often express a feeling of uneasiness when methods other than the textbook - way are suggested.

Some teachers feel that mathematics can only be learned in a setting which is rigid and organized. What kind of teacher - education courses, and what kind of inservice education would be needed to help teachers try new approaches to the teaching of mathematics? What kind of setting at the college or university level would result in positive transfer?

Three different groups and three different settings were used in this study. These groups differed with respect to many of the variables which were considered. The subjects of the teacher-directed group came from an area of high socio-economic status and their mean intelligence was higher than the intelligence means for the other two groups. How would these subjects react to a self-directed setting? What would the learning outcomes be for these subjects?

Three different treatments were used as part of the study. Is one of the treatments more effective than another? Is the effectiveness of a method related to specific topics which are to be considered? Is one of these methods more effective for one age group than another?

The results from the self-directed group seem to indicate that when students work in such a setting, for the first few weeks at least, pupil characteristics other than those selected for this study are related to success. What is the nature of the characteristics that may be related to success? What would be the relationship between the variables considered in this study and the learning outcomes if students are subjected to a self-directed setting over a long period of time?

Additional studies are required to determine the influence of particular learner, teacher, content and environmental characteristics on different modes of instruction.

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A P P E N D I C E S

APPENDIX A

INITIAL LEARNING AND RETENTION TEST

INITIAL LEARNING AND RETENTION TEST

School ______________________

Name ______________________

Directions

Read each question carefully and decide which of the four answers is the correct answer. Write the letter for this answer on the answer sheet beside the corresponding question number.

Do not write on the test.

Attempt all of the items.

Sample Items

I. Which of the following numbers has the largest value?

a) 23 b) 9 c) 35 d) 11

II. Which sign means add?

a) −

b) +

c) x

d) ÷

PART A

1). In the numeral 12.2 the value of the 2 in the ones' place is:

a) 100 times the value of the 2 in the tenths' place

b) 10 times the value of the 2 in the tenths' place

c) $\frac{1}{10}$ of the value of the 2 in the tenths' place

d) the same as the value of the 2 in the tenths' place

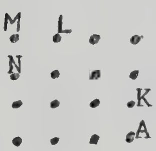
2). Which row has .6 of its objects shaded?

a)
b)
c)
d)

3). If you changed 3.20 to tenths, how many tenths would there be?

a) 2 b) 30 c) 32 d) 320

4).



If (4,1) is the number pair for the point A,

then (1,4) should be the name for point

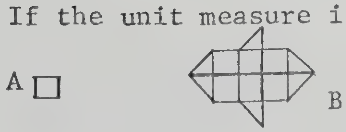
a) K b) L c) M d) N

5). A figure drawn on an array touches only the points

(1,1) (5,2) (5,5) and (1,5). How many sides does the figure have?

a) 3 b) 4 c) 5 d) 6

6). If the unit measure is square A, how many units are contained in figure B?



a) 9½ units b) 9 units c) 8½ units d) 8 units

7). Consider the array:

$$\begin{array}{cccccccccc} \vdots \\ 31 & 32 & 33 & \ldots \\ 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 \\ 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}$$

What number is meant when we write $7\uparrow\nearrow\rightarrow$?

a) 17 b) 27 c) 28 d) 29

8). 

If the distance between contour lines is 50 feet, the height of the hill shown would be:

a) exactly 200 feet

b) more than 200 feet

c) less than 200 feet

d) 100 feet

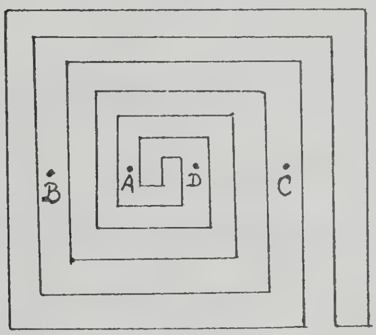
9). On a map, the Representative Fractions or R.F. is 1:31,400. This means that:

a) 1 inch on the map is 31,400 inches on the earth

b) 1 foot on the map is 31,400 feet on the earth

c) 1 mile on the map is 31,400 miles on the earth

d) 1 inch on the map is 31,400 miles on the earth

10). 

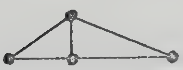
The points in the interior region of the closed curve shown are:

a) A and B

b) B and C

c) B and D

d) A and C

11). 

How many segments are there in the network shown?

a) 6 b) 3 c) 5 d) 4

12). If two dice are rolled and the two numbers showing are added, how many different sums are possible?

a) 11 b) 12 c) 2 d) 10

13). If two dice are rolled a large number of times and the two numbers showing are added each time, which prediction would best describe the outcomes?

a) Each possible sum would occur about one-sixth of the time.

b) Each possible sum occurs exactly the same number of times.

c) The sum of 7 will appear more often than any other sum.

d) The sum of 2 will appear less often than the sum of 12.

PART B

14). Another way to express .402 is:

a) .4 + .02

b) .4 + .01 + .01

c) .4 + .2

d) .4 + .002

15). What would be the effect if you dropped the zero from 3.90?

a) The number would have the same value.

b) The number would be one-tenths as large.

c) The number would be ten times as large.

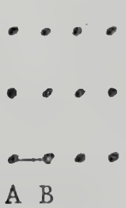
d) The number would be a hundred times as large.

16). If, on an array, you join the points (1,1) (5,5) and (10,10) the result would be a

a) triangle b) square

c) line segment d) closed curve

17).



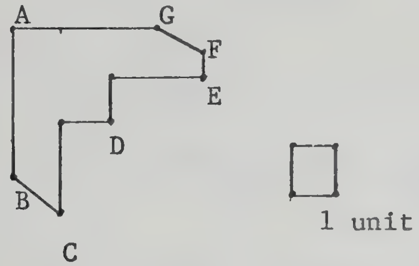
If the distance from point A to point B is one unit, then the shortest distance from (3,2) to point (7,2) would be:

a) 7 units b) 4 units c) 3 units d) 10 units

18). How should the decimal nine and thirteen-tenths be written?

a) 9 b) 10.3 c) 9.13 d) .913

19). The area of the region ABCDEFG is:



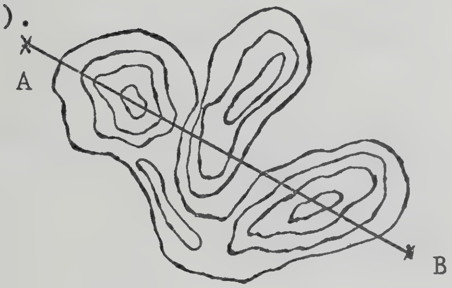
a) 5 units

b) 6 units

c) 7 units

d) more than 7 units

20).



If you plan to hike from A to B you would have to climb:

a) two peaks

b) three peaks

c) four peaks

d) more then four peaks

21). On a map the scale shown is: "1 inch to 2 miles". The distance between two towns is 3.5 inches. How far apart are the towns?

a) 3.5 miles b) 2 miles c) 6 miles d) 7 miles

22). If you multiply 8 x 45 and write the partial products as 32 and 40, then the 32 represents:

a) 3 tens and 2 ones

b) 3 hundreds and 2 tens

c) 32 ones

d) 3 thousands and 2 hundreds

23). The product of 5 x 56 is:

a) 25 ones and 3 ones

b) 25 ones and 30 ones

c) 25 tens and 3 tens

d) 25 tens and 30 tens

PART C

24). The problem .5 x .3 can be thought of as one-half of three-tenths. If you multiply these decimal fractions what would the answer be?

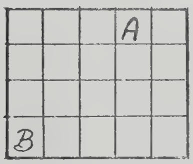
a) 1.5 b) .25 c) .015 d) .15

25). If you change .20 to a fraction with numerator 1 you will get:

a) $\frac{1}{5}$ b) $\frac{1}{200}$ c) $\frac{1}{20}$ d) $\frac{2}{10}$

26). Which decimal fraction has the smallest value:

a) 0.050 b) 0.500 c) 0.005 d) 0.550

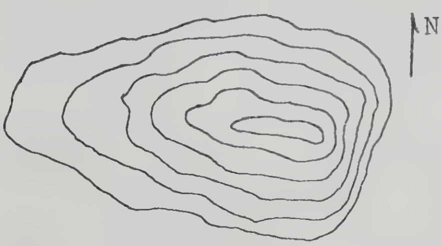
27). 

If the number pair (2,1) is assigned to the square labelled A, what number pair should be assigned to the square labelled B?

a) (1,1) b) (4,4) c) (4,5) d) (5,4)

28). If you subtract 2.4 from 71 your answer will be:

a) 68.6 b) 69.6 c) 4.7 d) 73.4

29). 

The steepest slope of the hill shown is on the:

a) north side b) south side

c) east side d) west side

30). If two coins are flipped a large number of times, which prediction would best describe the outcomes?

a) The outcome 2 heads occurred most often.

b) The outcome 2 tails occurred most often.

c) The outcome 1 head and 1 tail occurred most often.

d) The outcomes: <u>2 heads</u>; <u>2 tails</u>; <u>1 head and 1 tail</u>; occurred the same number of times.

A P P E N D I X B

DEFINITIONS OF THE COMPONENTS OF CALIFORNIA TEST OF PERSONALITY

DEFINITIONS OF THE COMPONENTS

PERSONAL ADJUSTMENT

1A. Self-reliance--An individual may be said to be self-reliant when his overt actions indicate that he can do things independently of others, depend upon himself in various situations, and direct his own activities. The self-reliant person is also characteristically stable emotionally, and responsible in his behavior.

1B. Sense of Personal Worth--An individual possesses a sense of being worthy when he feels he is well regarded by others, when he feels that others have faith in his future success, and when he believes that he has average or better than average ability. To feel worthy means to feel capable and reasonably attractive.

1C. Sense of Personal Freedom--An individual enjoys a sense of freedom when he is permitted to have a reasonable share in the determination of his conduct and in setting the general policies that shall govern his life. Desirable freedom includes permission to choose one's own friends and to have at least a little spending money.

1D. Feeling of Belonging--An individual feels that he belongs when he enjoys the love of his family, the well-wishes of good friends, and a cordial relationship with people in general. Such a person will as a rule get along well with his teachers or employers and usually feels proud of his school or place of business.

1E. Freedom from Withdrawing Tendencies--The individual who is said to withdraw is the one who substitutes the joys of a fantasy world for actual successes in real life. Such a person is characteristically sensitive, lonely, and given to self-concern. Normal adjustment is characterized by reasonable

freedom from these tendencies.

1F. Freedom from Nervous Symptoms--The individual who is classified as having nervous symptoms is the one who suffers from one or more of a variety of physical symptoms such as loss of appetite, frequent eye strain, inability to sleep, or a tendency to be chronically tired. People of this kind may be exhibiting physical expressions of emotional conflicts.

Social Adjustment

2A. Social Standards--The individual who recognizes desirable social standards is the one who has come to understand the rights of others and who appreciates the necessity of subordinating certain desires to the needs of the group. Such an individual understands what is regarded as being right or wrong.

2B. Social Skills--An individual may be said to be socially skillful or effective when he shows a liking for people, when he inconveniences himself to be of assistance to them, and when he is diplomatic in his dealings with both friends and strangers. The socially skillful person subordinates his or her egoistic tendencies in favor of interest in the problems and activities of his associates.

2C. Freedom from Anti-Social Tendencies--An individual would normally be regarded as anti-social when he is given to bullying, frequent quarreling, disobedience, and destructiveness to property. The anti-social person is the one who endeavors to get his satisfactions in ways that are damaging and unfair to others. Normal adjustment is characterized by reasonable freedom from these tendencies.

2D. Family Relations--The individual who exhibits desirable family relationships is the one who feels that he is loved and well treated at home, and who has a sense of security and self respect in connection with the various members of his family. Superior family relations also include parental control that is neither too strict nor too lenient.

2E. School Relations--The student who is satisfactorily adjusted to his school is the one who feels that his teachers like him, who enjoys being with other students, and who finds the school work adapted to his level of interest and maturity. Good school relations involve the feeling on the part of the student that he counts for something in the life of the institution.

2F. Community Relations--The individual who may be said to be making good adjustments in his community is the one who mingles happily with his neighbors, who takes pride in community improvements, and who is tolerant in dealing with both strangers and foreigners. Satisfactory community relations include as well the disposition to be respectful of laws and of regulations pertaining to the general welfare.

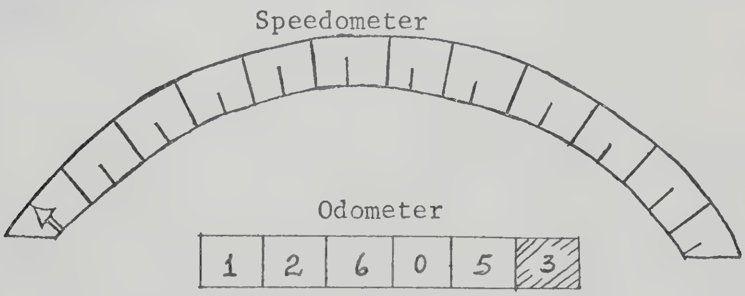
APPENDIX C

ACTIVITY BOOKLET

DISCOVERING DECIMALS

Fractions, or fractional numbers, are usually written with two symbols. For example a three (3) and a four (4) can be used to write $\frac{3}{4}$ (three-fourths). However, fractions can also be written in a different way, called decimal fractions. As you look at the following example, try to think of a reason for giving them this name.

Near the speedometer on your dad's car is the odometer which indicates how many miles the car has travelled.



Let's pretend the odometer on your dad's car shows these numerals. If you ask your dad to read this numeral he might say, "Twelve thousand, six hundred and five point three miles" or "Twelve thousand, six hundred five and three-tenths miles". On a piece of paper he would write:

$$12,605.3 \text{ or } 12,605\frac{3}{10}$$

Both numerals mean the same thing. For the first one he used a point - called a decimal point - to separate the whole numbers and the fractional numbers.

Write decimal fractions for:

1. The next number to appear on the odometer above  ______

2. The largest number that can appear on the odometer  ______

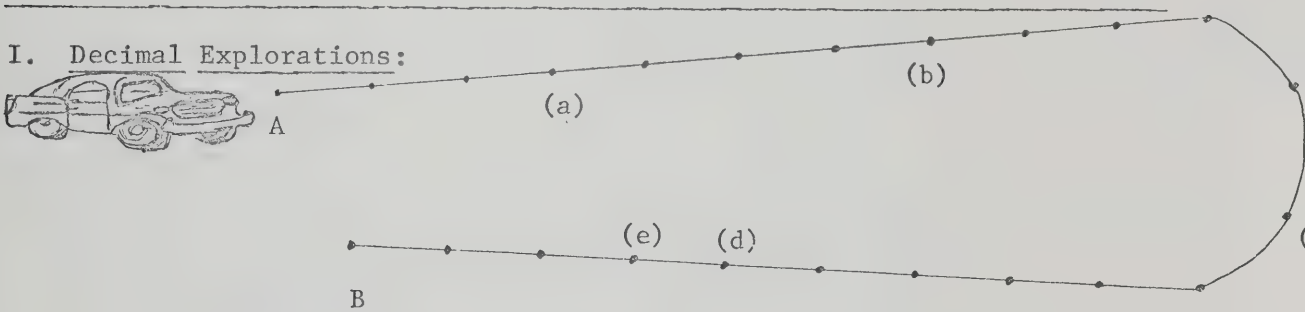
3. The smallest number that can appear on the odometer  ______

4. The number that would appear after driving twelve-thousand, six-hundred eight and seven tenths miles  ________

Why do you think they are called decimal fractions? (Compare your answer with a friend. Do you both agree?) ____________________

____

I. Decimal Explorations:



(1) Imagine that the line shown above is a road. The distance between two adjacent points is exactly one tenth (.1 or $\frac{1}{10}$) of a mile. The car travelling along this road is fairly new and its odometer shows | 0 | 0 | 2 | 8 | 3 | 2 | or 283.2 miles.

Write a decimal fraction for the odometer reading after the car has passed points:

(a) ________ (b) ________ (c) ________

(d) ________ (e) ________

What would the odometer reading be when it arrives at point B?

____

Make up similar problems for a very old car and a new car and let one of your friends give the answers.

*(2) Take a piece of rope or string (about an arm's length) and divide it into ten equal pieces. Mark your points of division with tape or crayon.

Get together with a friend. Use the length of the string as a unit of measurement. Take turns to estimate the distance between two points to the nearest tenth. Write down your estimates in decimal fractions and then measure the distance to see who gave the best estimate.

	ESTIMATES:		MEASUREMENTS:
	Yours	Friend's	
Your desk to his desk:	_______	_______	_________
Your desk to the door:	_______	_______	_________
His desk to the door:	_______	_______	_________
The door to a window:	_______	_______	_________

(3) Divide this square into ten equal pieces so that each piece is .1 or $\frac{1}{10}$ of the whole square.

a. Shade three of the pieces blue. What part of the whole square is blue? Write the answer as a decimal fraction.

____

b. Shade five of the pieces red. Write a decimal fraction for the red part.

____

c. Write a decimal fraction for the red and blue parts together.

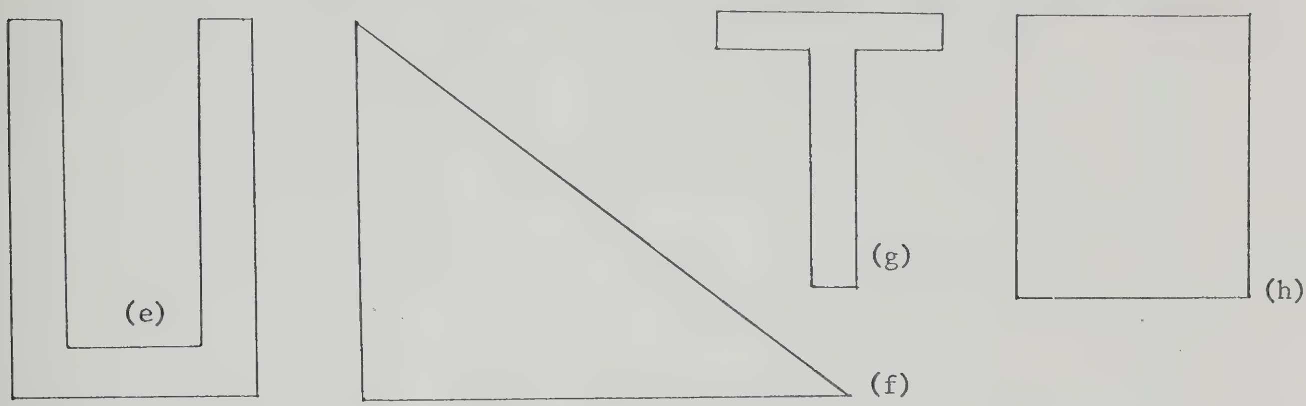
____

Compare your answers for parts a, b, and c with a friend.

d. Imagine the figures (see p. 4) are taken from, or cut out of the square above. Estimate how many tenths each figure is of the whole square. Write your estimate as a decimal fraction.

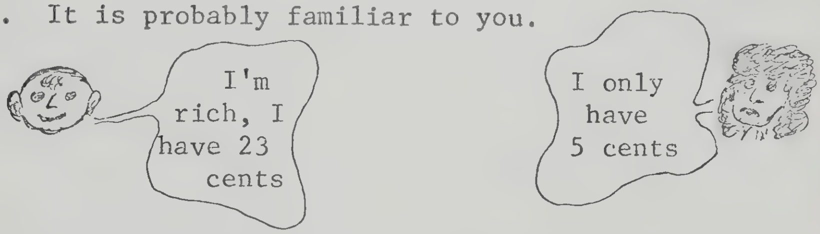
To get the measurements trace the figures or cut them out and fit them onto the square. (You may cut and reshape the figures if necessary). Compete with a friend to see who can give the best estimate.

	e.	f.	g.	h.
Estimated	______	______	______	______
Measured	______	______	______	______



II. You have probably used and written decimal fractions. Look at the example below. It is probably familiar to you.

Joe



Sue

Joe's amount can be written as 23 cents of 23¢ or \$ .23 or $\frac{23}{100}$ of a dollar.

Sue's amount can be written as: 5 cents or 5¢ or \$ .05 or $\frac{5}{100}$ of a dollar.

(1) Take 100 pennies (or counters and pretend they are pennies). Write decimal fractions for:

a. One handful of pennies ______

b. Another handful of pennies ______

c. The two handfuls of pennies ______

d. One of the pennies ______

e. All of the pennies ______

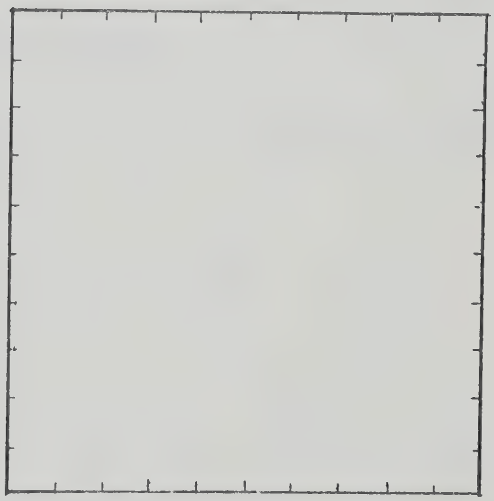
f. Suppose you have 26 pennies or \$ .26

What part of a dollar does the 2 represent? ______

What part of a dollar does the 6 represent? ______

(2) Beside each letter below (a to h) a pair of decimal fractions is written. Decide which one is larger and draw a circle around it. Explain to your friend why you think it is larger. Do you both agree? (You could pretend they represent money).

a.	.1	____	.01	e.	.09	____	.1
b.	.24	____	.27	f.	1.1	____	.11
c.	.33	____	.3	g.	.5	____	.55
d.	.99	____	.1	h.	.05	____	.5



III. Divide this square into one hundred equal pieces. Then each piece will represent .01 or $\frac{1}{100}$ of the whole square.

(1) Shade 13 of the pieces blue. What part of the square is blue? Write the answer as a decimal fraction. ____________

How would you read this decimal fraction?

____

(2) Shade 27 of the pieces red. What fraction of the square is red? ____________

How would you read this decimal fraction?

____

(3) What fraction of the square is blue and red? ________________

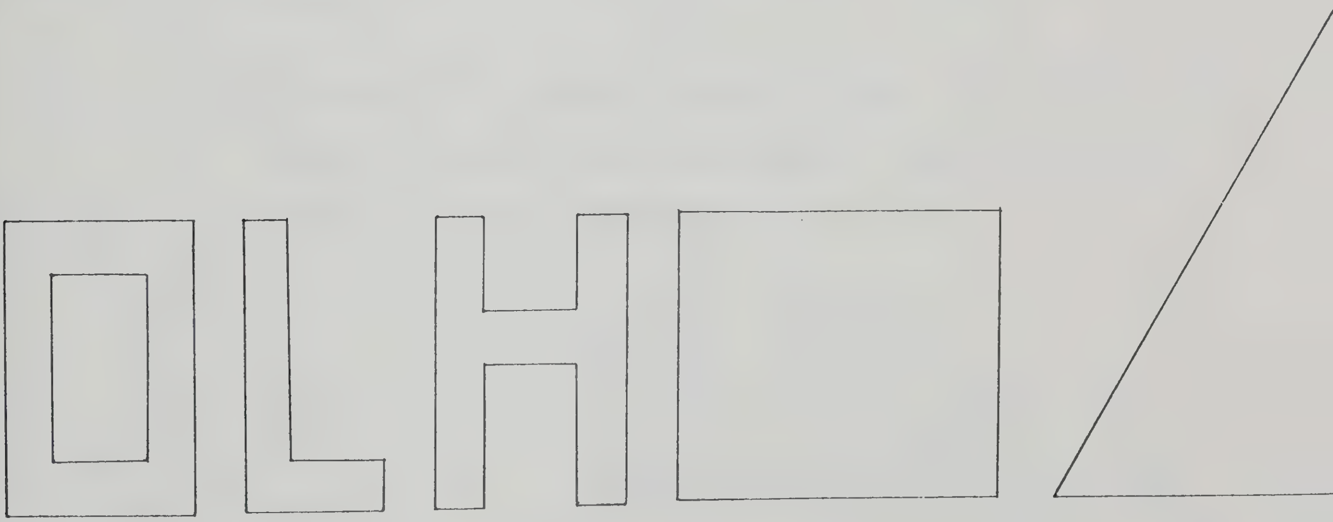
(4) Read the following decimal fractions. Use other colors to shade the parts of the squares expressed by the following decimal fractions:

(a) .05 (b) .20 (c) .1 (d) .99 (e) .50

(5) Imagine that the figures drawn below are taken, or cut out of the square above. Estimate how many hundredths each figure is of the whole square and write a decimal fraction. Find the best answer by tracing the figures, or cutting them out, and fitting them onto the square.

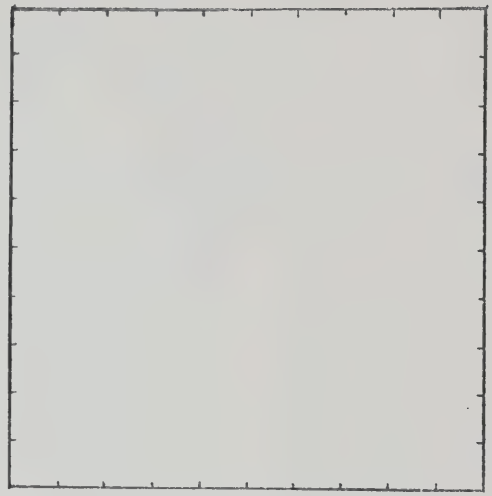
Compete with a friend to see who can give the best estimate.

	a	b	c	d	e
Estimated	______	______	______	______	______
Measured	______	______	______	______	______



(a) (b) (c) (d) (e)

*IV Divide this square into one hundred equal pieces.



(1) What part of the whole square above is each letter (see page 8). Write your answers in decimal fractions to the nearest hundredth. Use any way you like to find the answers.

T = ________ H = ________ E = ________

L = ________ I = ________ A = ________ F = ________

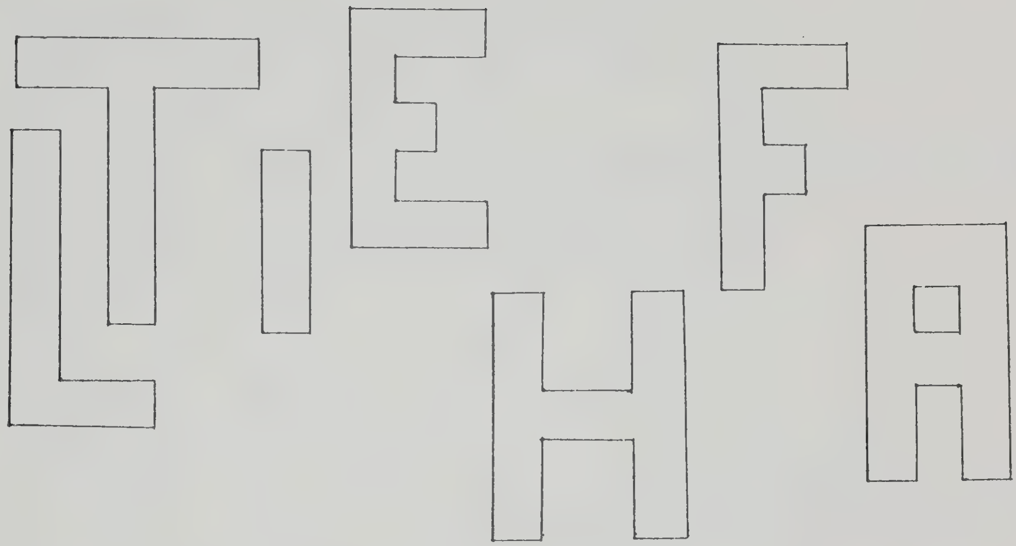
(2) Join any of the letters to make up words. Then determine what part of the whole square each word is. Check you answer by adding the decimal fractions.

Join T = ________

H = ________

E = ________

Count ________ Check ________



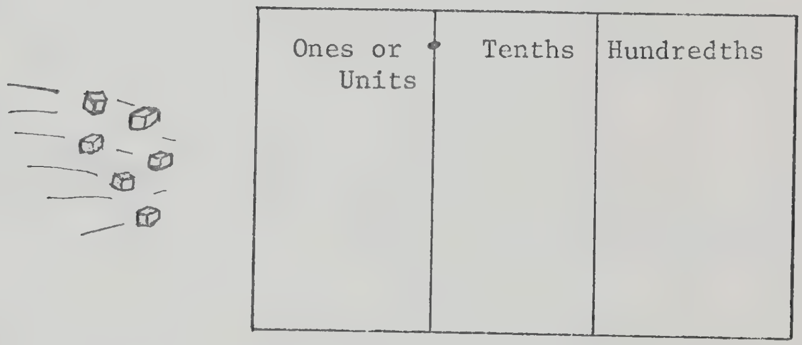
Can you make up a little decimal game to play with one of your friends?

V. ROLLING DECIMALS

Materials: Ten or more little blocks

Pencil and paper

A piece of paper or cardboard, 9 inches by 6 inches, divided into three equal pieces:



(1) Get together with one or two of your friends. Ask each to bring a pencil and paper. Place the marked paper or cardboard on a table or on the floor.

To begin, take ten blocks and roll them out on the marked piece of paper. As soon as they are rolled, cover them up with a large sheet of paper. Then each one writes down an estimate of the number the blocks now represent, if each block represents 1 in the units part, .1 in the tenths part, and .01 in the hundredths part. To see who gave the best estimate, remove the paper, check the blocks on the different parts, and make a count. Devise a way of keeping score.

Once you are familiar with the game, play it with more than ten blocks. Then you may have to regroup to give the correct number.

(2) Use the paper and the blocks as an adding machine. Roll five blocks. Record the number they represent. Remove the blocks and roll five more. Record this number. Calculate the sum of the two numbers. Use the blocks to check your answer.

Use the blocks and try to solve the following equations:

(a) $1.24 + .63 = N$

$N =$ ____

(b) $.59 + .43 = N$

$N =$ ____

(c) $4.62 + 3.51 = N$

$N =$ ____

(d) $.07 + .69 = N$

$N =$ ____

(3) How would you solve the equations:

(a) $3 \times .2 = N$ and (b) $5 \times .3 = N$?

NUMBER PAIRS

(______ , ______) On the first line write your weight to the nearest pound. On the second line write your height to the nearest inch. (If you do not know your exact weight and height, take a guess.)

You have written two numbers, or a Number Pair. Two things are important about such a Number Pair: - the order in which the numbers are written, and the meaning of each one. Look at the number pair your friend has written. Do you know his weight and height? How do you know?

There are all kinds of interesting things that can be done with number pairs. Try some of these:

1). Number Pairs can be used to write dates:
(day of the month, number of the month)

Write: today's date (_____,_____) yesterday's date (_____,_____)

tomorrow's date (_____,_____) your birthday (_____,_____)

2). Number Pairs can be used to compare two different groups:
Write a number pair for:

a. the number of boys and the number of girls in one row.
(_____,_____)

b. the number of boys and the number of girls in your room.
(_____,_____)

3). Number Pairs can be used to write riddles.
For example: (2,3) → 6. The two numbers of the pair have been multiplied to get 6.
Can you figure out what has been done to the members of each number pair?

a. (5,6) → 11

b. (8,3) → 5

c. (12,4) → 3

*4). Write some Number Pair Riddles of your own and let your friend try to solve them.

5). Each of the little squares can be referred to by a number pair.

Rows

5	E	J	O	T	Y
4	D	I	N	S	X
3	C	H	M	R	W
2	B	G	L	Q	V
1	A	F	K	P	U
	1	2	3	4	5

Columns

The square with the X could be called (5,4).
What does the 5 of the number pair refer to? ____________________

What does the 4 of the number pair refer to? ____________________

Using the same notation, what number pair names would you give to these squares?

A__________ D__________ G__________

H__________ K__________ N__________

Q__________ R__________ Y__________

6). a. Draw a circle inside each of the squares for which the same numbers are used. For example: (1,1)

b. Draw a triangle inside each of the squares for which the second number is twice the first one.

*c. Color red the squares that have two even numbers.

*d. Color blue the squares indicated by two odd numbers.

Compare your answers for questions 5 and 6 with a friend.

Treasure Hunt

1). Four treasures have been hidden. They can be located by following these directions:

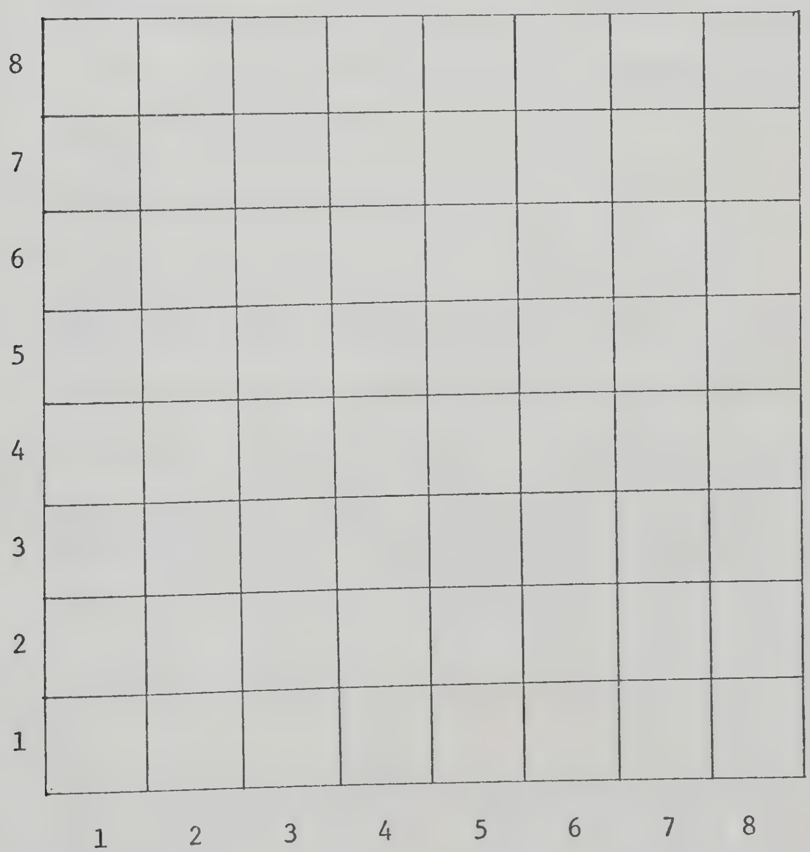
(a). Make a dot in the middle of the following squares:
(3,4) (3,7) (7,4) and (7,7)

With a straight - edge (ruler) join the dots in:
(3,4) and (7,4); (7,4) and (7,7); (7,7) and (3,7);
(3,7) and (3,4). This figure is called a ______________

(b). Make a dot in the middle of the following squares:
(1,4) (5,8) and (5,1).
With a straight edge join the dots in:
(1,4) and (5,8); (5,8) and (5,1); (5,1) and (1,4).
This figure is called a __________.

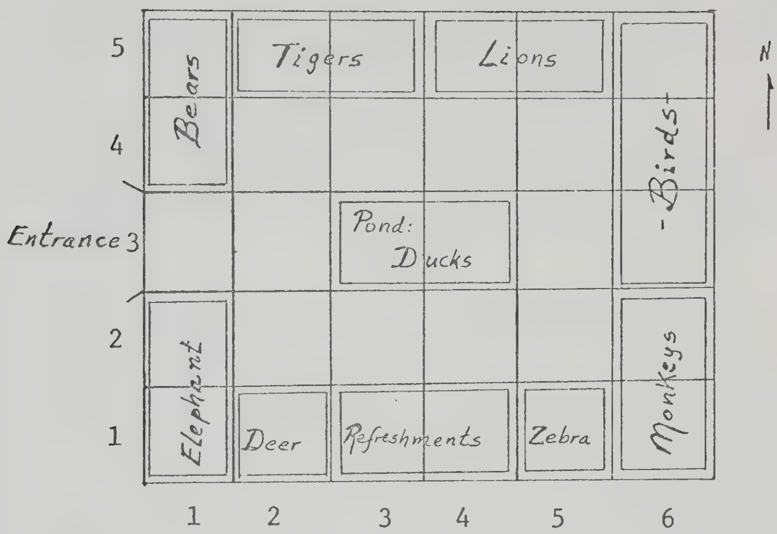
(c).. The treasure is hidden where a line drawn in (a) crosses a line drawn in (b).

The number pairs for these squares are:
(____,____), (____,____), (____,____) and (____,____).



MAPS

1). Here is a map of a zoo:



(a). One day someone left all the gates open and all the animals escaped. The zoo - keeper and his helpers rounded them all up and returned them to their cages.

Write the number pairs that locate the cages for the:
Bears (___,___) and (___,___) Monkeys (___,___) ; (___,___)

Lions (___,___) and (___,___) Elephant (___,___); (___,___)

Deer (___,___)

Zebra (___,___)

Write number pairs for:

(b). The place where popcorn could be bought (___,___).

(c) The location of the gate (___,___).

(d). The location of the pond (___,___) and (___,___).

(e). How many number pairs would you have to write for the sidewalk that had to be swept after closing? ________

*2). The map below is divided into squares. From the information given draw in the following:

(a). A river flows through the middle of (1,3) (2,3) (3,3) (4,3) into (5,3); turns north into (5,4) (5,5) (5,6); and turns east into (6,6) and (7,6).

(b). A sports center is located in (1,6) (1,7) (2,6) and (2,7). The football field runs east and west and the goals are in (1,7) and (2,7). The baseball diamond can be found in (2,6) and the swimming pool in (1,6).

(c). Most people live in the south eastern corner. There is a church in (7,1) and a school in (5,2).

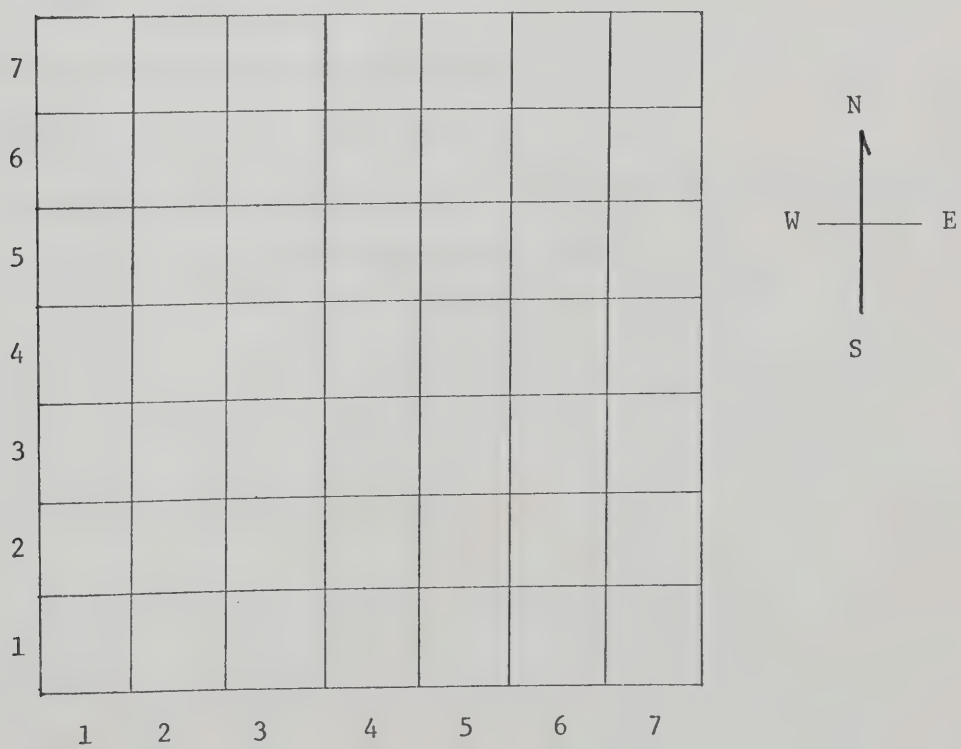
(d). A straight road runs from the middle of (5,1) to the middle of (1,5). Where would the bridge across the river have to be? ________

(e). A straight railroad track runs from the middle of (4,1) through the middle of (5,7). In which square does the railroad bridge cross the road? ________________

(f). A swamp is located in (1,1) (1,2) (2,1) and (2,2).

(g). A small lake with a lot of trees around it can be found in (4,5) and (4,6).

(h). Some hills can be found in (7,3) (7,4) and (7,5).

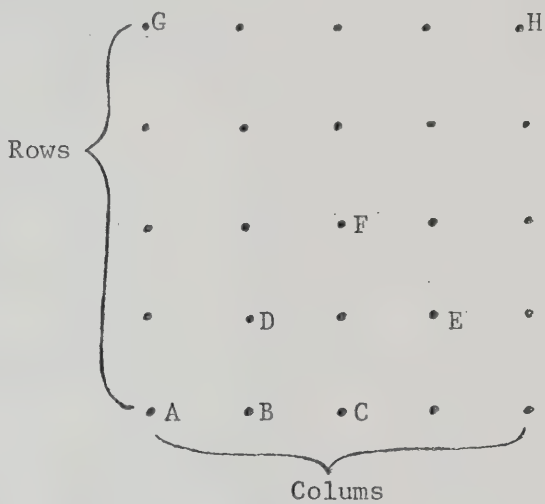


MORE NUMBER PAIRS

Instead of using squares, let's assign a number name to each dot of the array shown.

Dot A we will call (1,1) because it is in column 1 and row 1.

Dot B we will call (2,1) because it is in column 2 and row 1



1). Using the same notation, what number names would you give to the posts labelled:

C (____,____)   D (____,____)

E (____,____)   F (____,____)

G (____,____)   H (____,____)

2). For how many dots on the array are the first and last part of the number name the same? For example (3,3) ________

Do these dots make a pattern? __________

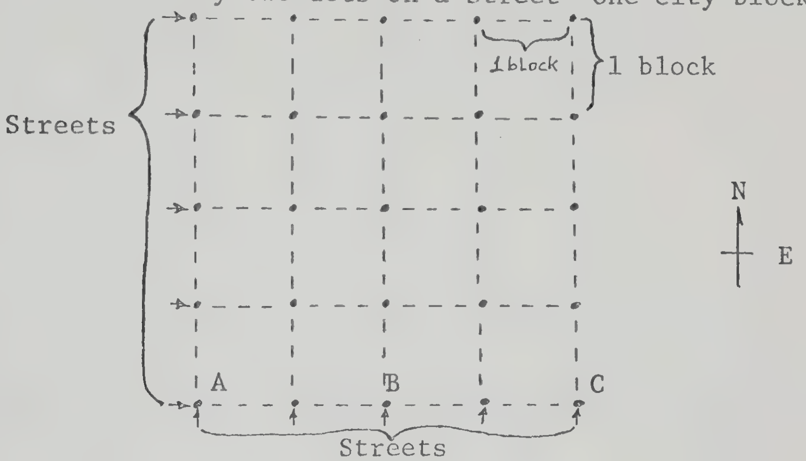
3). For how many dots is the sum of the two parts of the number name equal to six? __________.

*4). For how many dots on the array are both parts of the number name a. even? ________ b. odd? ________

*5). For how many dots on the array is the first part of the number name twice as large as the second part of the name? __________

DISTANCE AND NUMBER PAIRS

Let's pretend the columns and rows on the array are city streets. Call the distance between any two dots on a street one city block.



1). Look at the array. Pretend you are walking from street corner A to street corner B. To get from A to B you could take many different routes.

What is the shortest distance in blocks from A to B? ______________

The number name for street corner A is: (____,____)

The number name for street corner B is: (____,____)

Can you find a way of telling the distance of the shortest route from A to B by making a calculation from the number names for A and B? (Hint: B is 2 blocks east of A. How does this show in the number name?)

How did you do it? Explain it to a friend.

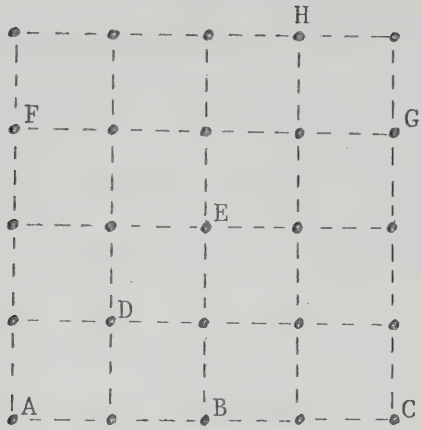
2). Let's walk from A to corner C.

The number name for A is: (____,____)

The number name for C is: (____,____)

Can you work out how many blocks the shortest route from A to C is by using the number names for A and C? Take a guess and check your answer.

Did you guess correctly? ______________



3). Try your method for other routes. Guess the shortest route by looking at the number names. Check your guess on the array.

From corners:			Guess	Check
C to D:	(____,____)	(____,____)	_____blocks	_____blocks
A to E:	(____,____)	(____,____)	_____blocks	_____blocks
B to F:	(____,____)	(____,____)	_____	_____
A to H	(____,____)	(____,____)	_____	_____
C to F:	(____,____)	(____,____)	_____	_____

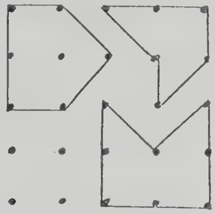
Did you find a way of guessing the shortest distance that always works? ____________

Compare your answer with a friend.

* Make up two questions of your own and let your friend try to answer them.

FIGURES

Talk about the following questions with a friend and figure out the answers.

1). 

- A -

Look at the figures shown on array A. Can you find two things these three figures have in common? What are they? ____________________

2). Draw a four - sided figure on array B. Do you know a name for this figure? Can you draw another 4 - sided figure that looks different?

- B -

3). On array C, draw a three - sided figure. Do you know a name for this figure? Can you draw another 3-sided figure that looks different?

- C -

4). On array D, try to draw a six-sided figure. Can you think of anything that looks like the figure you have drawn? Try to draw a figure with more than six sides.

-D-

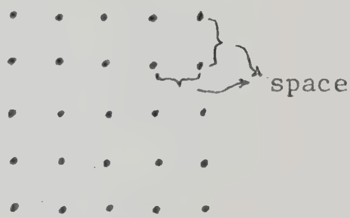
5). A drawn figure touches only the dots (3,1) (2,3) and (4,4). Try to draw this figure. How many sides does the figure have? __________.

6). A figure touches only the dots (2,2) (3,1) (3,3) and (4,2). Guess how many sides the figure has: ____________ Draw the figure and check your guess.

FIGURES AND FENCES

Let's pretend each of the dots on the array is a fence post. Let's call the distance between any two posts in a column or row 1 space and use number pairs as you did on the previous pages.

1). Build a fence by drawing on array A a line around the posts (4,4) (4,5) (5,4) and (5,5). Pretend the inside or interior of this figure is a garden. How long is the fence around the garden? ________ spaces.



- A -

2). Another garden has a fence around the following corner posts. (4,3) (4,5) (5,3) (5,5). Draw this garden on array B. How long is the fence around this garden? ____________ spaces.

- B -

3).

- C -

A larger garden has a fence around the posts (4,2) (4,5) (5,2) and (5,5). Draw this garden on array C. How long is the fence for this garden? ________________

4). All these gardens (Questions 1 to 3) are located in columns 4 and 5. One more garden that is even larger than the last one can be drawn in these two columns. Name the four corner posts for this garden.

(___,___) (___,___) (___,___) (___,___) Guess the length of the fence around this garden: ____________

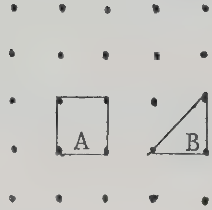
Draw this garden on array D and check your guess.

- D -

5). Use any part of array E. Draw fences only along the rows and columns and see how many different shaped gardens you can draw that have a fence exactly 12 spaces long. Compare your answers with a friend.

- E -

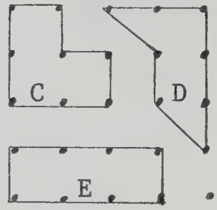
You have drawn fences around gardens. Now let's look at the inside or interior of the gardens. Let's call the size (area) of garden A "1 square".



1). If the area of garden A is one square, what would the area of garden B be? ______________. How do you know?

2). Look at the gardens C, D, and E. Can you find one thing these three gardens have in common? ___________

How do you know?



3). On arrays A and B, draw as many gardens as you can with an area of 5 squares. Compare your answer with a friend and prove to him that your gardens have an area of 5 squares.

- A -

- B -

4). On arrays C and D, draw as many gardens as you can with an area of 8 squares. Compare answers and prove to your friend that your garden has an area of 8 squares.

— C —

- D -

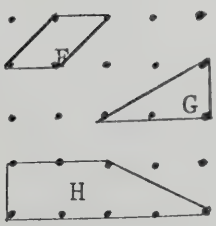
5). On array E, try to draw gardens that have an area of:

a. $1\frac{1}{2}$ squares

b. $5\frac{1}{2}$ squares

- E -

6).



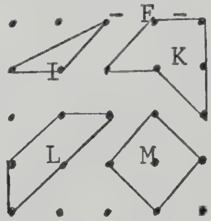
- F -

Look at arrays F and G and try to find the area for the gardens shown: F______ I________

G______ K________

H ______ L________

M

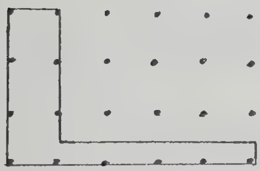


- G -

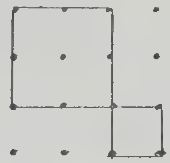
Compare and discuss your answers.

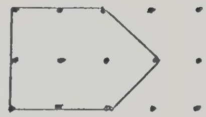
HOW MANY SQUARES?

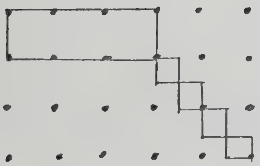
Try to find the area for the figures shown on this page. Compare answers and explain how you found your answer.

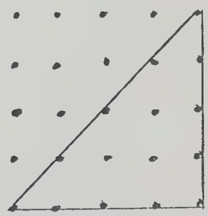


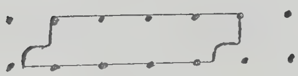
___ Squares



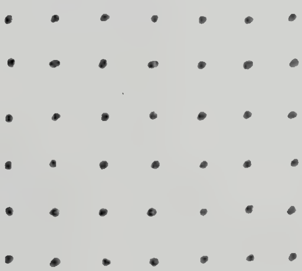


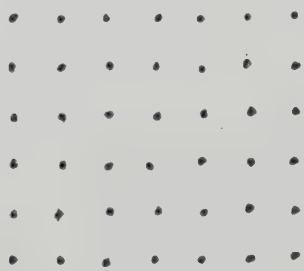






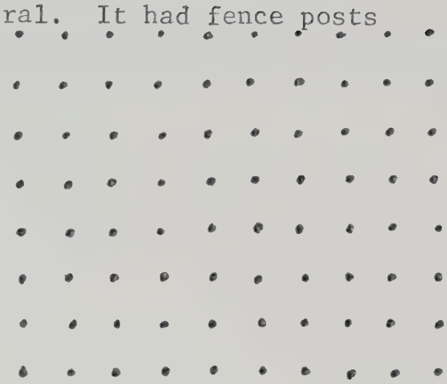
* Draw some figures of your own. Have a friend find the area for these figures.





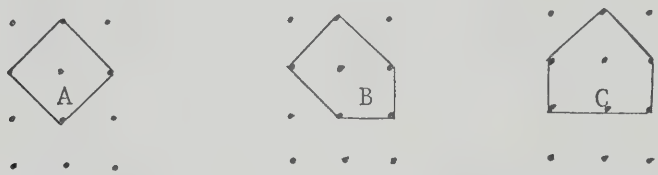
AREA AND FENCE POSTS

Cowboy Pete had a very strange corral. It had fence posts arranged in an array like this:



Pete could fence off sections simply by wrapping a rope around the posts.

One day he decided to fence off corrals that had one post in the interior or inside:



1). For corrals A, B, and C count the number of fenceposts.

Figure out the area for each one and record your results in the table.

Corral	Number of Posts	Area ('squares')
A		
B		
C		

2). Suppose you were to fence off a similar corral with 7 fence posts. Guess the area of the corral: _____ squares. Draw the corral on the array above and check your guess. Record your result.

3). Guess the areas of corrals that are fenced off with 8, or 9, or 3 fence posts. Check your guess. Compare and record your results.

You have used arrays of squares and dots for different activities. Let's write numbers in an array.

.									
.									
.									
21	22	.	.	.					
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Now this gives a new way to write names for numbers:

1). What number do you suppose is meant when we write:

$3 \rightarrow$? ________

Does your friend agree with your answer? How did you decide on the number?

2). What number is meant when we write $7\uparrow$? ________.

3). Try to find simpler names for each of these numbers?

a. $8\rightarrow$ ________ b. $9\leftarrow$ ________ c. $5\uparrow\uparrow\uparrow$ ________

d. $3\nearrow$ ________ e. $9\leftarrow\uparrow\rightarrow$ ________ f. $21\downarrow\rightarrow$ ________

g. $3\uparrow\uparrow\uparrow\uparrow\uparrow$ ________ h. $3\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow$ ________ i. $24\swarrow$ ________

j. $27\rightarrow\leftarrow\rightarrow\leftarrow$ ________ k. $27\rightarrow\leftarrow\rightarrow\leftarrow\uparrow$ ________ l. $27\swarrow\rightarrow\uparrow\rightarrow$ ________

*4). Write some names for numbers in the new way. Let your friend try to write simpler names for these numbers.

*5). What simpler names would you write for the following numbers? Discuss your answers with your friend.

a. $1\leftarrow$ ________ b. $10\rightarrow$ ________ c. $20\nearrow$ ________

d. $11\swarrow$ ________ e. $9\rightarrow\rightarrow$ ________

6). For the operation of addition (+), the sum of 0 and any number is the number itself. That is why it is so easy to solve:

4 + 0 = ______ 0 + 36 = ______ 1,232 + 0 = ______

<u>Zero</u> is called the <u>Identity</u> <u>for</u> <u>addition</u>.

7). Make a true statement for the following:

_____ x 4 = 4 36 x _____ = 36 1,232 x ______ = 1,232

What number is the <u>Identity</u> <u>for</u> <u>multiplication</u>? ______________

8). Which of these sentences are identities? Take a guess and then use any numbers to check your guess.

a. ________ →← ________

b. ________ ↗↓→ ________

c. ________ ↗↓← ________

d. ________ ↗↗↗ ↓↓↓ ← ← ________

e. ________ ↗↗↗ ↓↓↓ ← ← ← ________

f. ________ ↑↓↑↓↑↓↑↓ → → ↙↙ ↑↑ ________

9). Try to make up some identity sentences of your own.

a. 5 = 5

b. 23 = 23

c. 10 = 10

* d.

* e.

* f.

NUMBER PAIRS AND DIRECTIONS

The signs ↑ and → could also be written in front of a number to locate points on a grid. Let's agree to start at zero. Point A in Figure 1 could be found by looking at the number pair (→3, ↑ 2). How is this done?

1). Try to find the point (→2, ↑ 3) on Figure 1. Call this point B.

2). Remember to start at zero, and graph or plot the point (→4, ↑ 4). Call the point C.

3). Graph point D at (→ 0, ↑ 5)

4). Is it possible to graph a point called (← 3, ↑ 1) on Figure 1?

Why or why not? ______________________________

Could you graph this point on Figure 2?

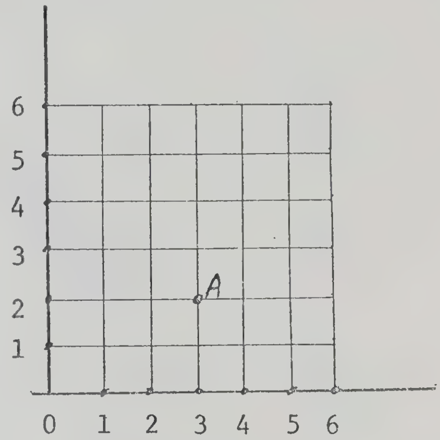


Figure 1

5). Using the same notation, how would you direct some-one to the following points:
A. (,   )
B. (,   )
C. (,   )
D. (,   )
E. (,   )
F. (,   )

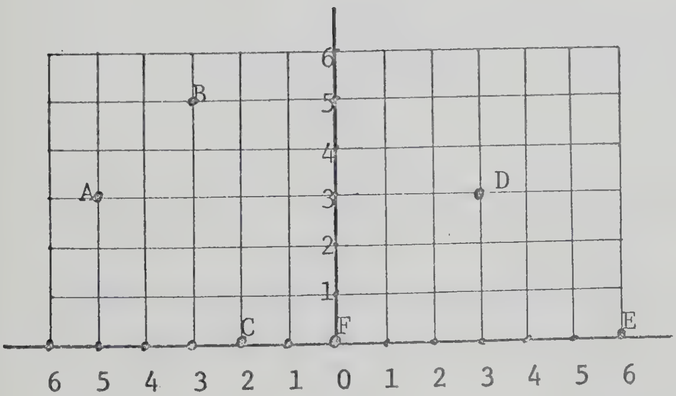


Figure 2

6). Some-one made up the following number pairs:
a. (← 5, ↓ 3)   b. (← 2, ↓ 2)   c. (→1, ↓ 1)   d. (→3, ↓ 4)

Is it possible to graph these points on Figure 2? ______
Is it possible to graph these points? How could this be done?

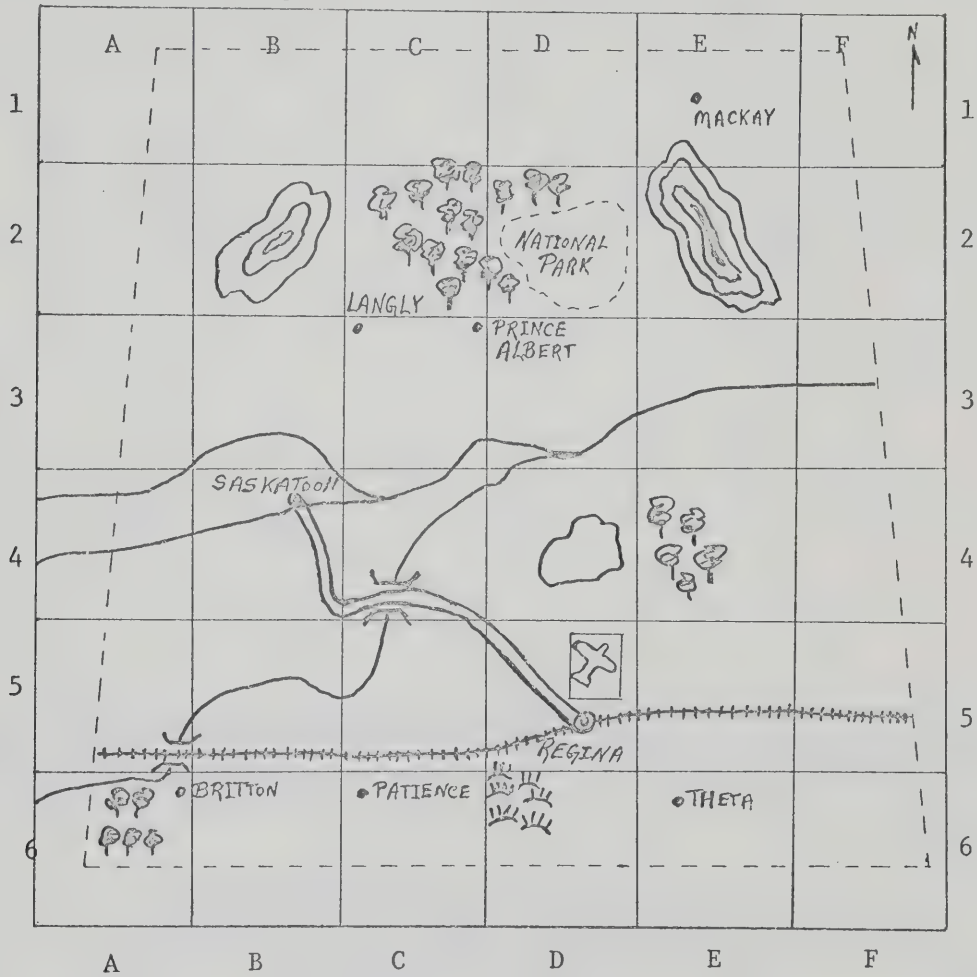
*7). Make up number pairs using this notation and see if your friend can find them. Check his answers.

a. (,   )   b.   c.

MAKING AND USING MAPS

Map-making and map-reading both require that we make use of mathematics. Numbers, number pairs, and operations are used in many different ways. A few examples follow.

1). LOCATIONS. Some maps use letters only, others use letters and numbers to make it easy to locate places.



Write down the location, using letters and numbers, of the following places:

a. Mackay (E,1) b. Britton _______ c. Langly _______

d. Prince Alberta_______ e. Saskatoon _______ f. Regina _______

g. Theta _______ h. Patience _______ i. National Park _______

j. Railroad Bridge __________.

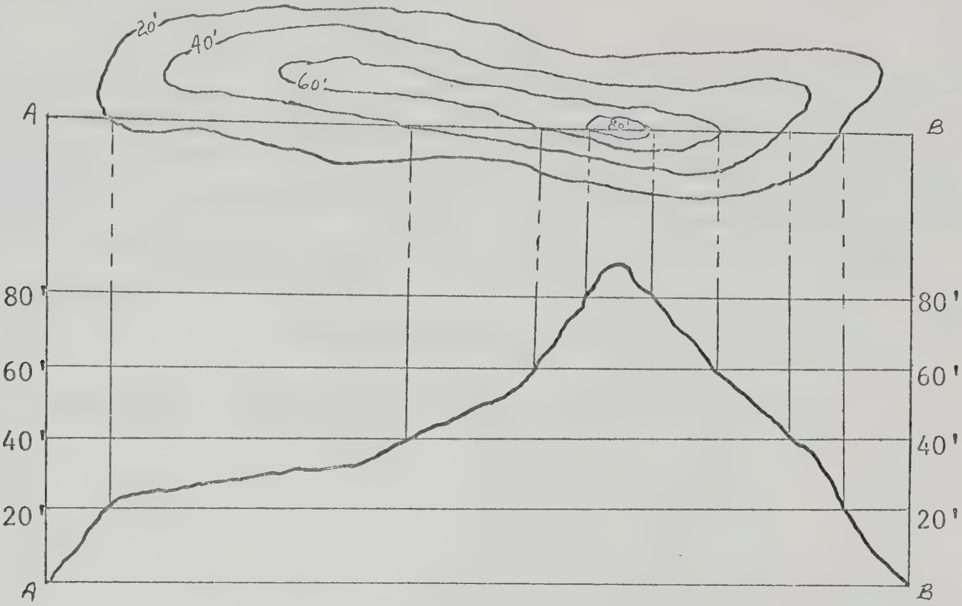
*2). Write down the name of a town, city, river,lake or mountain. Now locate it in your atlas. Look in the index. There the page number and a letter-number pair or a capital letter-small letter pair will be given. Compete with a friend to see who can find it first. Make a game out of it.

If you have tried to draw a hill or mountain, you know that it is rather difficult. Map-makers have invented a clever way of showing mountains on a map. With surveying instruments they measure the height or elevation of many points around the hill and then plot them, like this:

Probably this looks confusing. To make a clearer picture, take a pencil and join with lines all the points that have the same elevation. These lines are called Contour Lines. Your finished drawing should look something like this:

By looking at the countour lines can you tell how high the hill is? Is it exactly 60 feet high? Why or why not?

*What does this hill look like? Let's find out. Suppose you wanted to walk from point A to point B. What would your hike be like?



The lower part of the drawing shows a way of finding out what your climb might be like in going from A to B. Can you figure out how it was done? Explain it to one of your friends.

1). Look at this drawing, The distance between the contour lines is 100 feet.

Railroad   Building

There is a hill or mountain within a sight of Union Station.

How many peaks does the mountain have? ________________

What is the approximate elevation for the highest part of the mountain?

____

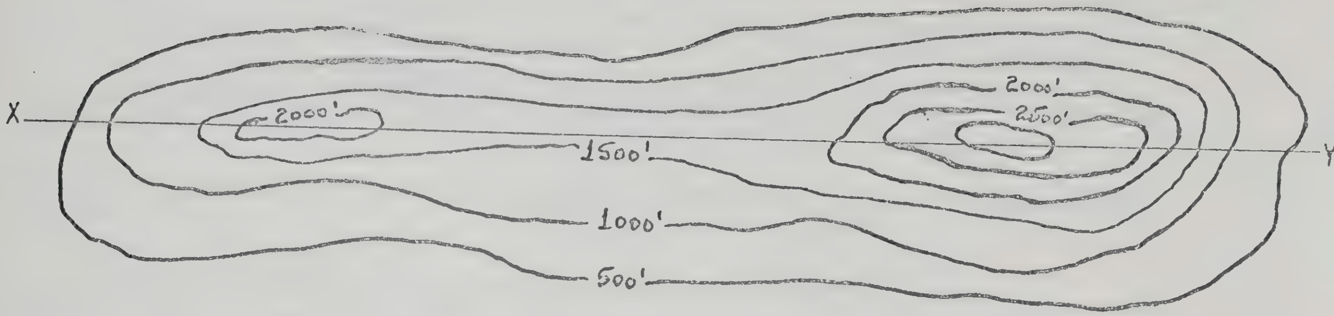
What is the approximate elevation of the island?

____

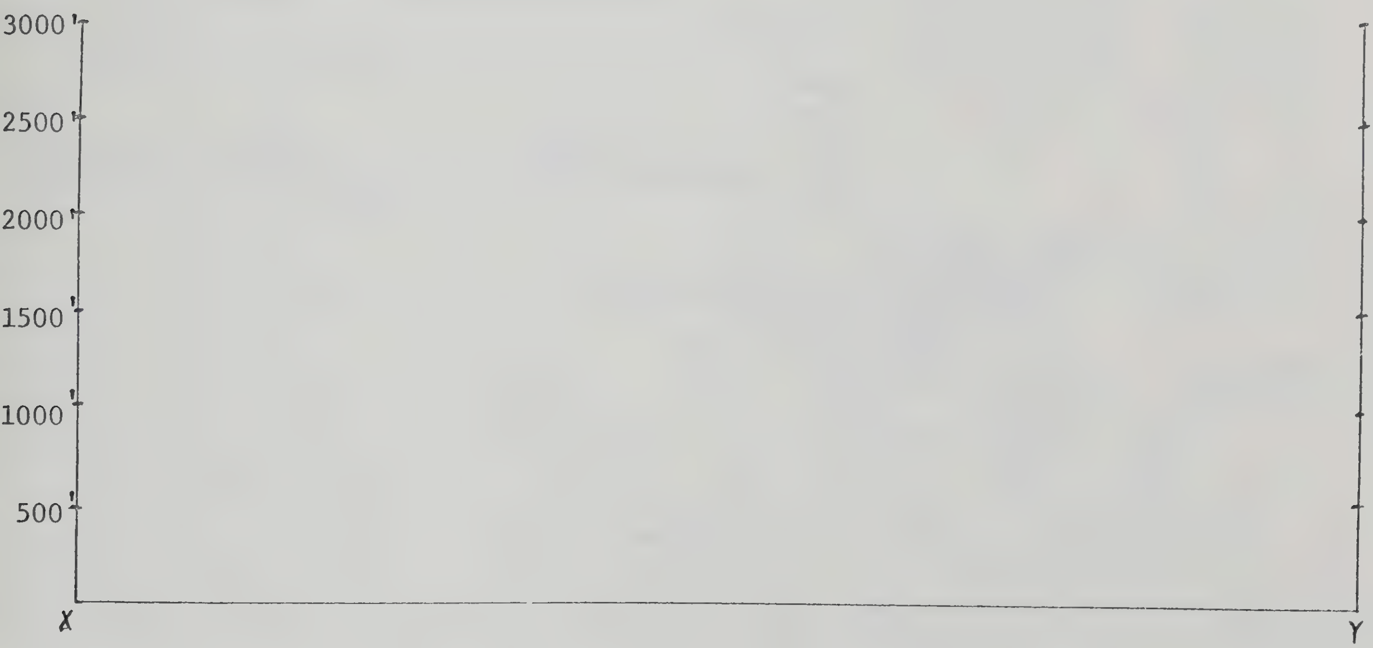
What is the difference (in feet) between the highest peak and the highest point on the island?

____

*2).



Use the example from page 30and make a drawing of the path from point X to point Y.

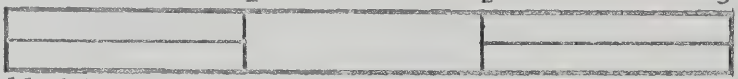


3). Draw an imaginary mountain or mountain range with contour lines. Provide the distances between the contour lines. Draw in a route for a climb or hike. Find out what the path of your hike might look like.

Maps are drawn to scale. Everything is drawn as it is in the real world, but made smaller. Somewhere on each map is a scale to indicate how much smaller the map is. The scale is usally shown in one of three different ways:

a.) By a statement: One inch equals 30 miles.

b.) By a bar graph:



c.) By a fraction, called Representative Fraction or R.F.: 1:50,000 which means that 1 inch on the map equals 50,000 inches on the earth.

Below is a small map of Banff, Alberta. The scale is shown with a bar graph.

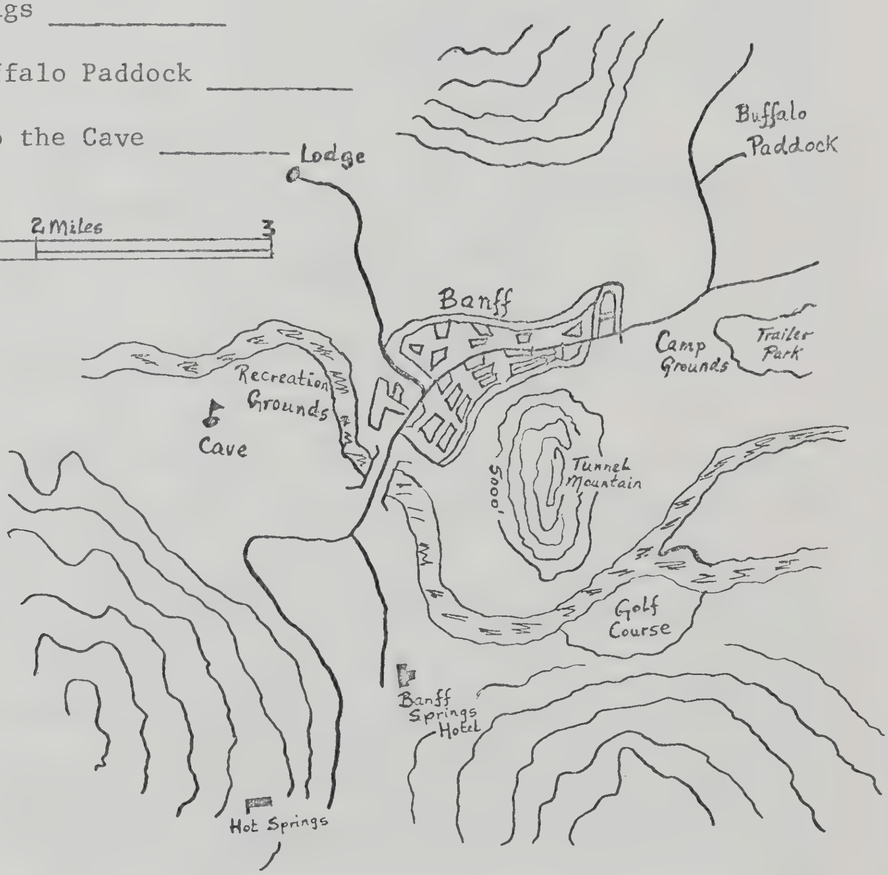
1). Use this scale and a piece of string or a strip of paper to find the approximate distance from:

a. Banff to the Lodge ____________

b. Banff to the Hot Springs ___________

c. Hot Springs to the Buffalo Paddock _________

d. Banff Springs Hotel to the Cave ________



2). If the distance between contour lines is 100', what is the approximate height of Tunnel Mountain? ______________________

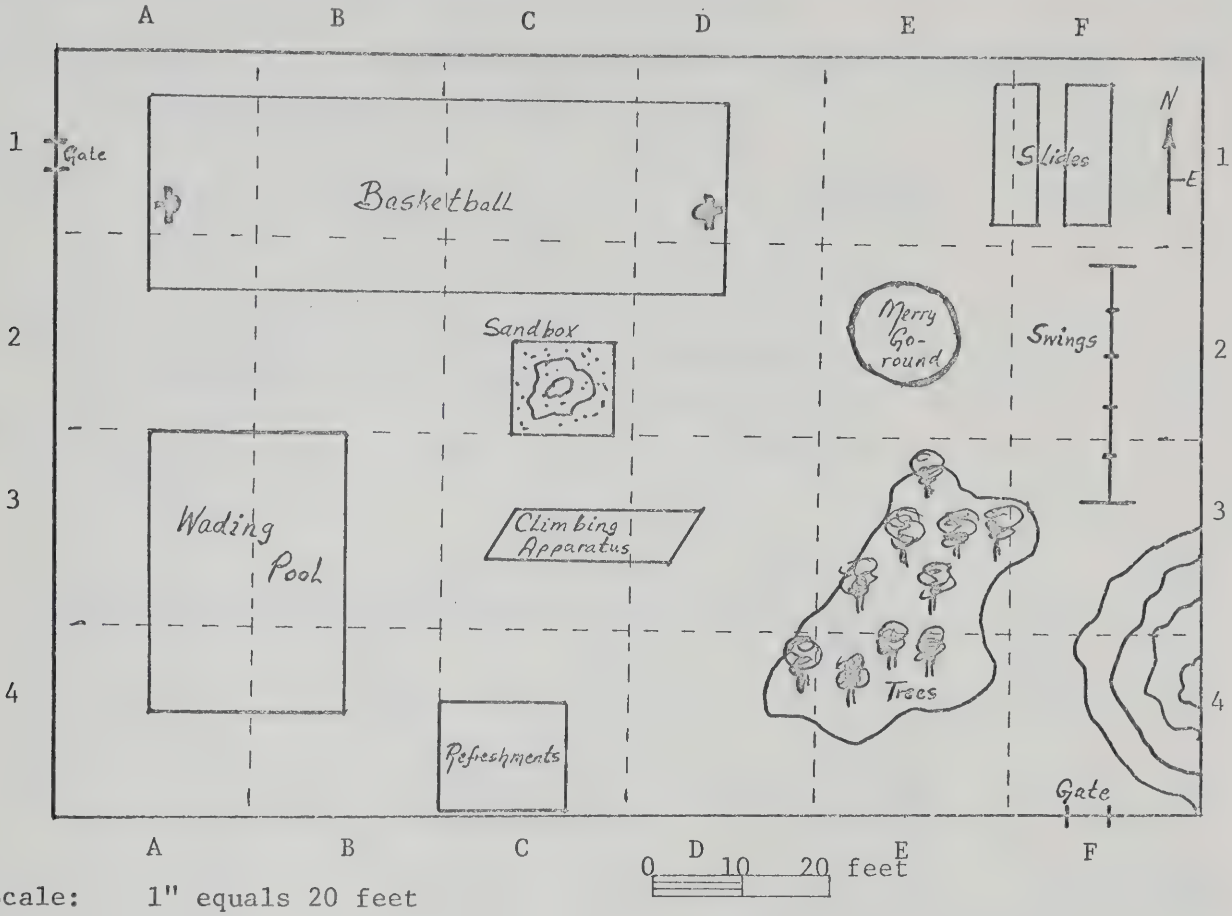
3). Use the R.F. 1:50,000 and a ruler to find these distances in inches:

a. Cave to the Hot Springs ______________

b. Banff to the Trailer Park ______________

c. Camp Grounds to the Hot Springs _______________

This is a map of a playground:



Scale: 1" equals 20 feet

1). If a friend wanted to meet you at (E,2) what might he be doing? ______________________

2). Your friend and you walked to (B,1). Why? ______________________

3). After an hour of hard play, you get thirsty. Where would you go to buy a drink? (,)

4). What direction would you take to get from the sandbox to the swings? ______________________

5). How large is the wading pool? Length:_____feet. Width:_______feet

6). How long are the slides? __________feet.

7). How far is it from one basket to the other? __________feet.

8). Approximately how far is it from the south gate to the west gate? __________

9). The distance between contour lines is 5 feet. How high is the hill in the southeast corner? __________feet.

10). A truck delived new sand. How high is the pile in the sand box? ________feet

11). Make up questions of your own and see if one of your friends can answer them.

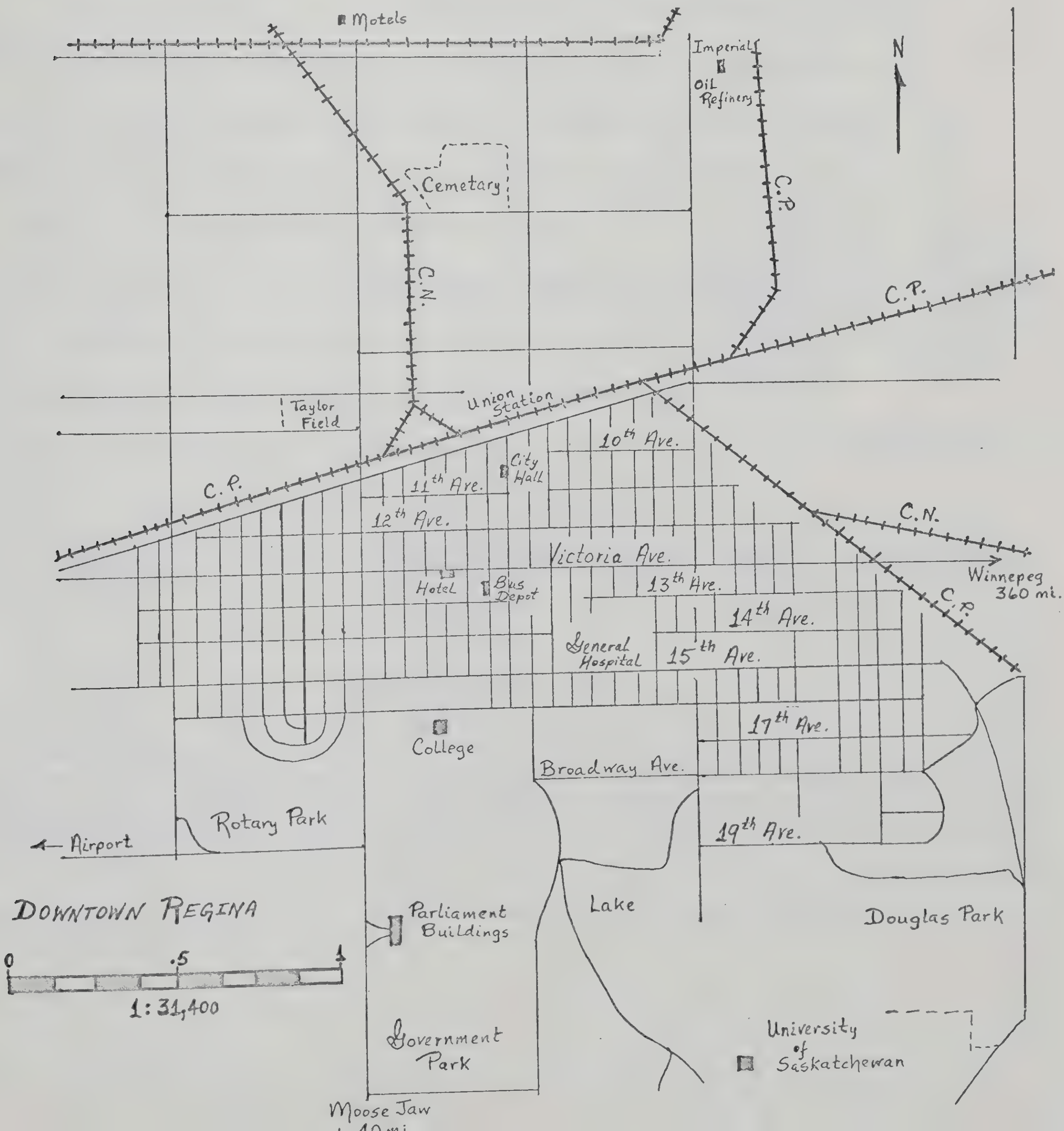
Here is a map of Regina, Saskatchewan. Use the map and the information of the scale to answer the following:

*1). a. Use a ruler and measure the length of the map: ______ inches.
b. Change your answer to miles: ______
c. Change your answer to feet: ______

*2). a. Measure the width of the map: ______ inches
b. Change your answer to miles: ______
c. Change your answer to feet: ______

*3). Find the distance in miles from:
a. Taylor Field to the Parliament Buildings: ______
b. Bus Depot to Union Station: ______

*4). Make up questions of your own and have a friend try to find the answers.



CURVES, NETWORKS AND REGIONS

CURVES: A Closed Curve divides any flat surface on which it is drawn into a number of Regions. For example, in Figure 1, the circle divides the region of this page (plane) into two regions, an Interior and an Exterior region.

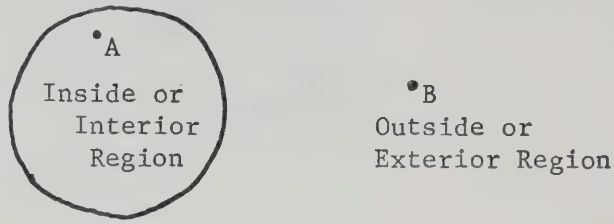


Figure 1

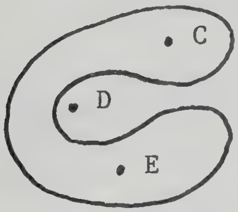


Figure 2

How many regions are there in Figure 2? ________
Shade in part of the exterior retion.

In figures 1 and 2 it is easy to tell whether any one particular point (A, B, C, D, or E) is located in an interior region or exterior region. Sometimes this is not so easy. Look at Figure 3. Are the points 1, 2, 3, 4, 5 and 6 in the interior or exterior region? Before you check it, do the following: Draw a straight line from the point you are considering to a point you know is in the exterior region (for example 2 to F) and count the number of times this segment intersects the curve. Record your answer in the table below.

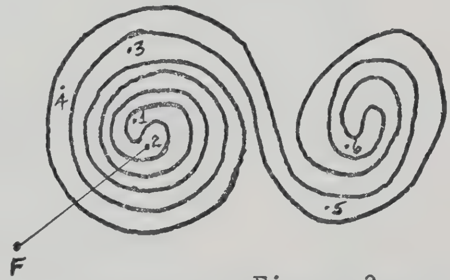


Figure 3

Points	No. of Intersections	Check: Interior or Exterior
1		
2	5	Interior
3		
4		
5		
6		

Look at the intersections you have counted. What kind of numbers match with points in the exterior region? What kind of numbers match with points in the interior region? Now can you state a rule for finding out whether a point is in the exterior or interior region? ________

*On a piece of paper draw a closed curve of your own and see if your rule works for your curve.

In a network you can see several points or junctions, lines or segments joining these points, and the various regions into which the plane represented by this page is divided by the segments.

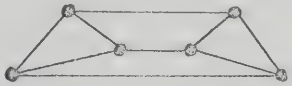


Figure 1

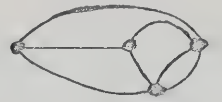


Figure 2

1). For the network shown in Figure 1, count the:
Segments: ________
Junctions: ________
Regions: ________ (Did you remember to count the exterior region ?)

2). For the network shown in Figure 2, count the:
Segments: ________
Junctions: ________
Regions: ________

3). Draw a network of your own, (Figure 3) Count the:
Segments: ________
Junctions: ________
Regions: ________

Figure 3

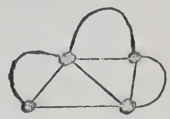


Figure 4

Figures 1, 2, and 4 on this page show Connected Networks. They consist of one part and on each network you can get from one junction to any other by going along the lines or segments of the network.

4). Is the network you drew (Figure 3) a Connected Network?

Note: You may be puzzled when a junction occurs in the middle of a line as in Figure 5 (a). Should we call this one line or two?

Figure 5:

(a) (b) (c) (d)

We decide that (a), (b), and (c) are all alike; in each of them we have 3 junctions and 2 segments. In (d) a line ends where it began. This we call 1 junction and 1 segment.

5). Try to draw a Disconnected Network.

CONNECTED NETWORKS:

1). Look at the networks shown in Figures 1 to 4. Count the number of junctions, regions and segments. Record your results in the table.

Figure 1

Figure 2

Figure 3

Figure 4

Figure	Junctions	Regions	Segments
1			
2			
3			
4			

2). For networks with two junctions, what can you say about the number of regions and segments? ________________

3). What number would you have to add to the 'number of segments' in order to make the following statement true?

(Number of Junctions) + (Number of Regions) = (Number of Segments) + ________

4). Count the number of junctions, regions, and segments for the networks shown in Figure 5 to 8. Record your results in the table.

Figure 5

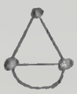


Figure 6

Figure 7

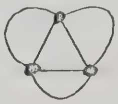


Figure 8

Figure	Junctions	Regions	Segments
5			
6			
7			
8			

5). Is your statement about the number of regions and segments for question 2 true for networks with three junctions?

6). Does your answer for question 3 hold for networks with three junctions? ________________

7). Count the number of junctions, regions and segments shown in the networks in Figure 1 to 4. Record your results in the table.

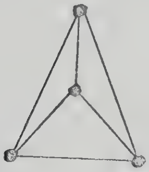


Figure 1

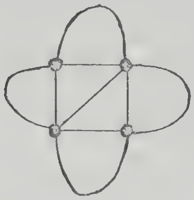


Figure 2

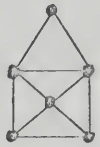


Figure 3

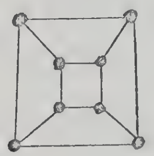


Figure 4

Figure	Number of Junctions	Regions	Segments
1			
2			
3			
4			

8). What can you say about your answer to questions 2 and 3 on the previous page when you are working with networks that have more than two junctions? ______________________________

____

9). Draw some connected networks of your own. Check to see what you can say about your answers to questions 2 and 3 for your own networks.

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

You are probably familiar with this chart. It is not very difficult to use and it summarizes the basic multiplication facts.

Find the answers for: 2 x 8 = __________ 8 x 8 = __________

6 x 7 = __________ 5 x 3 = __________

It looks as if the chart just contains the facts up to 9 x 9. If you look carefully you might also find the multiplication tables for 12, 13, ..., 19, ..., 23, ..., 45, ..., 123, and many more.

Let's look at multiplying by 12. Instead of separating the 1 and 2 in the top row, think of them as being together: the 2 is written in the ones place and all the ones can be found under the 2. The tens are written under the one.

1	2
tens	ones
0	0
1	2
2	4
3	6
4	8
5	10
6	12
.	.

Here is one example:

2 x 1 | 2 or 2 x 12 = 2 | 4 or 24

Find the answer for:

3 x 12 = ______________

Sometimes it will be necessary to regroup before giving the answer.

For example:

6 x 12 = 6 tens + 12 ones = 7 tens + 2 ones or 72

MULTIPLICATION CHART

1). From the chart find the answers for the following. (Check your answers if you are not sure):

7 x 12 = ____	2 x 34 = ____	2 x 67 = ____
8 x 12 = ____	5 x 34 = ____	5 x 67 = ____
9 x 12 = ____	8 x 34 = ____	9 x 67 = ____
2 x 25 = ____	3 x 48 = ____	2 x 19 = ____
5 x 25 = ____	6 x 48 = ____	6 x 19 = ____
7 x 25 = ____	9 x 48 = ____	9 x 19 = ____

2). Try these questions with larger numbers:

2 x 123 = ______	2 x 357 = ______
5 x 123 = ______	4 x 357 = ______

*3). Maybe you can discover a way to use the same chart for division. Try to find the answers for:

46 ÷ 23 = ____

60 ÷ 12 = ____

138 ÷ 23 = ____

170 ÷ 34 = ____

96 ÷ 12 = ____

4). 11 is not included on the multiplication chart. It's easy to write down the table numbers. Complete the following series:

11, 22, ____, ____, ____, ____, ____, ____, 99

Some numbers are easily multiplied by 11. Look at these examples:

$$\begin{array}{r} 45 \\ \times\ \underline{11} \\ 495 \end{array} \qquad \begin{array}{r} 32 \\ \times\ \underline{11} \\ 352 \end{array}$$

9 = 4 + 5 5 = 3 + 2

Multiply the long way and try to find out why this works.

$$\begin{array}{r} 63 \\ \times\ \underline{11} \end{array} \qquad \begin{array}{r} 72 \\ \times\ \underline{11} \end{array} \qquad \begin{array}{r} 54 \\ \times\ \underline{11} \end{array} \qquad \begin{array}{r} 27 \\ \times\ \underline{11} \end{array}$$

Number Puzzles

5	6	30
4	2	8
20	12	240

For the puzzle:

1. Multiply across 5 x 6 = 30

   4 x 2 = 8
2. Multiply the products

   30 x 8 = 240
3. Multiply down 5 x 4 = 20

   6 x 2 = 12
4. Multiply the products 20 x 12 = 240

1). See if these puzzles work:

3	2	
5	4	

6	7	
5	8	

9	12	
10	8	

21	6	
40	19	

*2). Complete the puzzles:

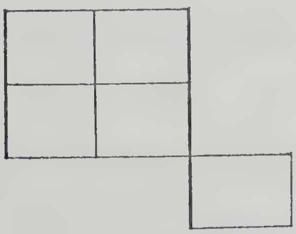
4		20
8	6	

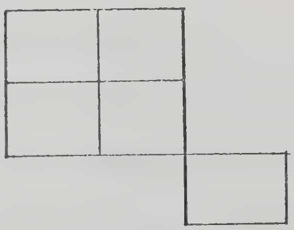
7	9	
		20

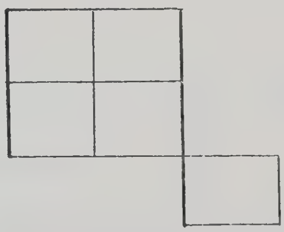
		80

6		42
		252

*3). Make your own puzzles:

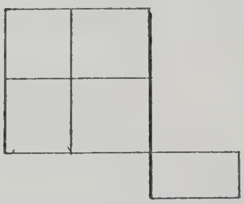


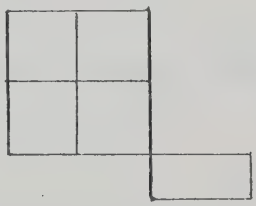




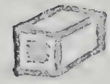
*4). Can you think of some numbers that cannot be used for these puzzles?

Try to think of some.





Why can't they be used? Do you know a name for these numbers?



SUGAR LUMP PROBABILITY

Mark one face of a sugar lump with a flow pen or with some paint.

1). What fraction of the time do you think the lump would land with the colored face up if you rolled it like a die? ________

2). Roll the lump 24 times and keep track of the number of times the lump lands with the colored face up.

Total	Colored Face Up

What fraction of the time did it land with the colored face up? ________

Do you need to change your guess?

3). Try it again. This time roll the lump 36 times. What fraction of the time did it land with the colored face up? ________

Total	Colored Face Up

4). Add up the total number of rolls and the total number of times the colored face came up for your group and then for the whole class.

Group		Class	
Total	Colored Face Up	Total	Colored Face Up

5). How many faces does the lump have? ________

6). Can you see a relationship between the number of faces and the fraction of times the colored face comes up? What is the relationship? ________

7). Now mark or paint 2 faces of a lump. What fraction of the time do you think this lump would land with the colored face up? ________

Total	Colored Face Up

Roll the lump 36 times and record your results. Record the results from your group and from the class.

Group		Class	
Total	Colored Face Up	Total	Colored Face Up

How do these results compare with the first ones you got?

ROLLING DICE

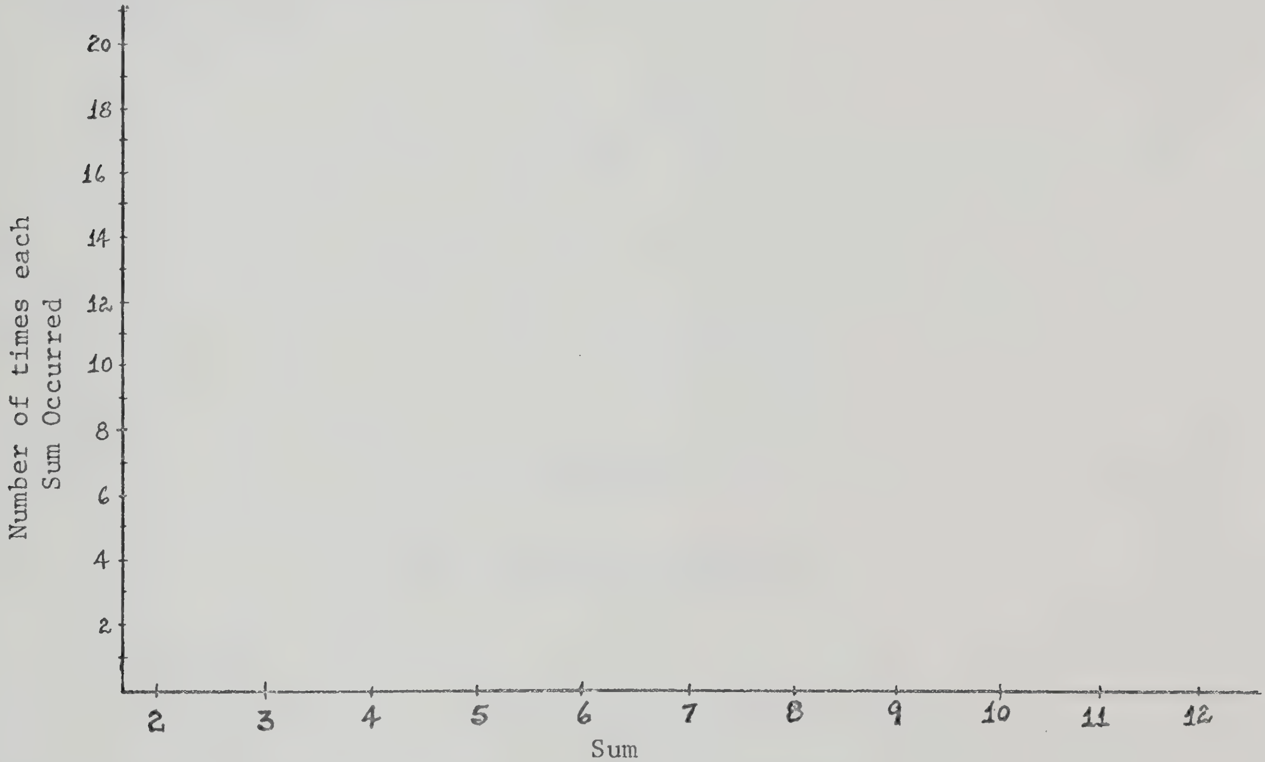
1). If 2 dice are rolled what are the possible sums of the faces that can come up?

2). Do you think each sum is equally likely to happen?

3). Roll the dice many times, say 72 times, and record the sum of the 2 faces each time.

4). Which sum occurred most often? ______________

5). Put your results on the graph by putting a dot to show how many times each sum occurred and then join the dots.



What fraction of the time did you get 7 as a sum? ____________

3? ____________ 11? ________________

Compare your results with your friends. Make a graph for the results from the whole class.

A P P E N D I X D

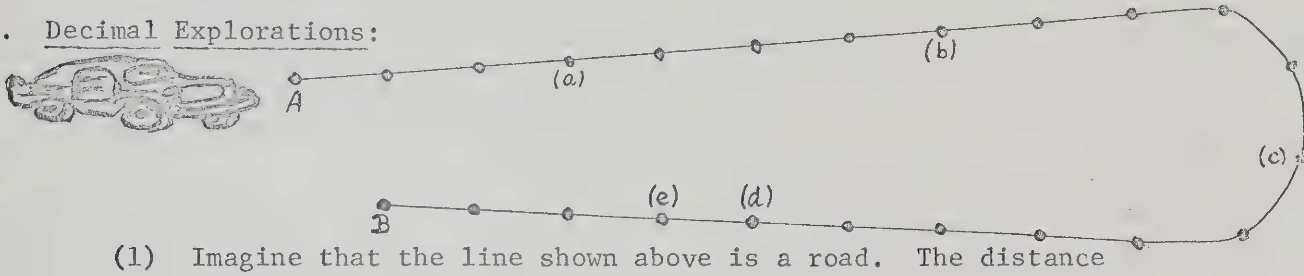
REVISED PAGES FOR TD-GROUP

4. The number that would appear after driving twelve-thousand, six-hundred, eight and seven tenths miles ______

Why do you think they are called decimal fractions? ______

______

I. <u>Decimal Explorations</u>:



(1) Imagine that the line shown above is a road. The distance between two adjacent points is exactly one tenth (.1 or $\frac{1}{10}$) of a mile. The car travelling along this road is fairly new and its odometer shows | 0 | 0 | 2 | 8 | 3 | 2 | or 283.2 miles.

Write a decimal fraction for the odometer reading after the car has passed points:

(a) ______ (b) ______ (c) ______

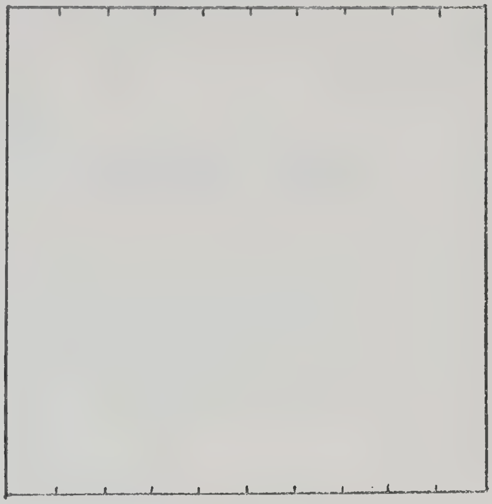
(d) ______ (e) ______

What would the odometer reading be when it arrives at point B?

______

Make up similar problems for a very old car and a new car.

(2). Divide this square into ten equal pieces so that each piece is .1 or $\frac{1}{10}$ of the whole square.



(a). Shade three of the pieces blue. What part of the whole square is blue? Write the answer as a decimal fraction. ____________

(b). Shade five of the pieces red. Write a decimal fraction for the red part. ____________

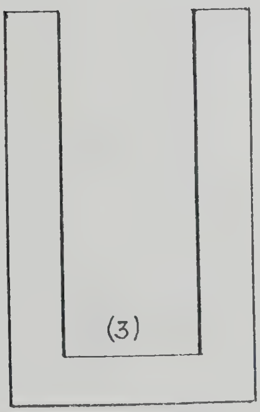
(c). Write a decimal fraction for the red and blue parts together.

____

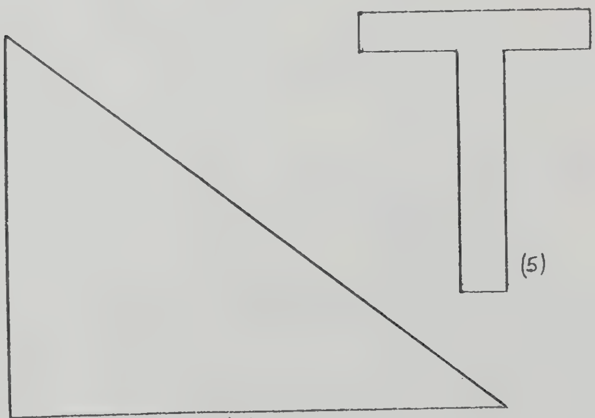
Imagine the figures below are taken from, or cut out of the square above. Estimate how many tenths each figure is of the whole square. Write your estimate as a decimal fraction.

To get the measurements trace the figures or cut them out and fit them onto the square. (You may cut and reshape the figures if necessary.)

	(3)	(4)	(5)	(6)
Estimated	______	______	______	______
Measured	______	______	______	______



(3)

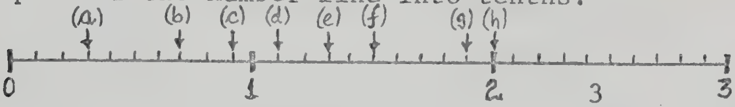


(4)

(5)

(6)

7. Look at the number line. The marks between 1 and 2 divide this part of the number line into tenths.



You can write the fraction numerals $\frac{3}{10}$ or .3 for mark (a). Write two fraction numerals for marks (b) and (c):

(b) ________ or ________ (c) ________ or ________

You can write the numerals $\frac{11}{10}$ or $1\frac{1}{10}$ or 1.1 for mark (d).

Write three numerals for marks (e), (f), (g), and (h):

(e) _____, _____, _____ (g) _____, _____, _____

(f) _____, _____, _____ (h) _____, _____, _____

8. Look at the pairs of decimal fractions below. Decide which one of the two is larger and circle it.

a. .3 - 3.0

b. 3.3 - 3.3

c. 0.2 - 1.2

d. .4 - 4.0

e. .2 - .3

f. 0.8 - 8.0

g. .9 - .8

h. 2.0 - 2.1

9. Write decimal fractions for:

a. $\frac{1}{5}$ or $\frac{2}{10}$ = ______

b. 1 7/10 = ______

c. 2 9/10 = ______

d. $\frac{1}{10}$ = ______

e. $\frac{10}{10}$ = ______

f. $5\frac{5}{10}$ = ______

10. Change the decimal fractions to fractions with denominator 10.

a. .3 = ______

b. 1.6 = ______

c. 6.4 = ______

d. 5.0 = ______

e. .8 = ______

f. 8.8 = ______

II. John rides to school on his bicycle. He travels five-tenths of a mile to pick up his friend. Then they travel one and seven tenths miles to get to school. How far is it from John's house to the school?

$0.5 + 1.7 = N$ You must find the numeral to replace N.

(a)
$$\begin{array}{r} {}^{1}\\ 0.5 \\ +\ 1.7 \\ \hline 2 \end{array}$$

Add the tenths' column. The sum is 12 tenths. Think of 12 tenths as 1 and 2 tenths. Write, 2 tenths in the tenths' place. Write 1 in the ones' column to show 1 mile.

(b)
$$\begin{array}{r} {}^{1}\\ 0.5 \\ +\ 1.7 \\ \hline 2.2 \end{array}$$

Write the decimal point to separate the ones from the tenths. Add the ones. The sum is 2.2 or 2 and 2 tenths miles.

Write an equation for the following problems and solve the equation.

Use decimal fractions.

1. Suppose an odometer of a car shows the following numerals:

5	2	6	2	4	5

What would the reading on the odometer be after the car travelled to another city twenty and nine-tenths miles away?

Equation: ______________

2. While Bob's dad was on a trip he bought five and four-tenths gallons of gas at one gas station and six and nine-tenths gallons at another station. How many gallons of gas did he buy?

3. One summer morning the temperature was seventy eight and seven-tenths degrees Fahrenheit. During the day it rose by thirteen and five-tenths degrees. What was the highest temperature of that day?

4. Find the answers to the following question. Use the number-line below to check your answers.

a. .8 + 1.6 = _____ c. 1.7 + 2.3 = _____

b. 2.4 + 1.8 = _____ d. 1.8 + 1.9 = _____

5. Make up some problems of your own. Give them to a friend and let him try to solve them.

III. Mary went shopping with her mother. She looked at a colored television set whose price tag read $999.99. Mary was told by her mother that each 9 in the price represented a number ten times as great as the 9 to the right of it. Is that right? How do you know?

$ 9 9 9 . 9 9

What part of a dollar does this 9 represent? ← → What part of a dollar does this 9 represent?

Why is the decimal point used on the price tag?

1. Write decimal fractions for:

   a. 23 cents: ______ { What part of a dollar does the 3 represent? ______ / What part of a dollar does the 2 represent? ______

   b. 5 cents: ______

   c. Two dollars and twenty five cents: ______

   d. Three dollars and three cents: ______

   e. One dollar and sixty three cents: ______

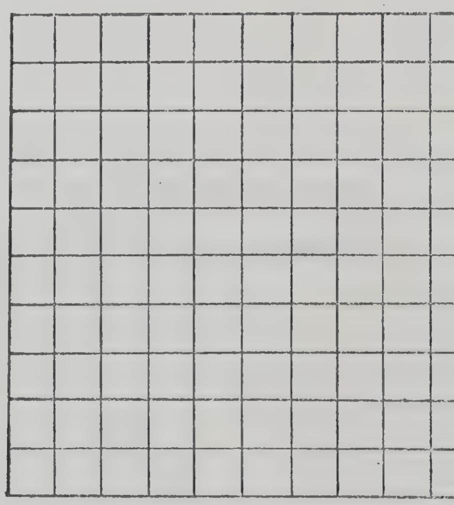
2. The numeral 2.12 can be read as "Two point twelve", "Two decimal twelve", or "Two and twelve-hundredths."

   Which of the three ways gives the most information about the numeral?

   Read the following decimal fractions:

a) 1.01	e) 4.06	i) 8.79
b) 5.55	f) 12.50	j) 9.63
c) 0.46	g) 12.05	k) .03
d) 4.60	h) 12.00	l) 100.10

3. Look at the square. How many equal pieces are there? ______



   a. Write a decimal fraction for one of the pieces. ______

   b. Shade in 23 of the pieces. Write a decimal fraction for the shaded part: ______

   c. Write a decimal fraction for the part that is not shaded in: ______

   d. Write a decimal fraction for all of the pieces: ______

4. Look at the pairs of decimal fractions below. Decide which of the two is larger and circle it.

a. .01 - .10	d. 5.04 - 5.10	g. 6.9 - .69
b. .8 - .79	e. 1.00 - .99	h. 8.08 - 8.80
c. .12 - 1.2	f. 1.6 - 1.08	i. 2.42 - 2.45

5. Write a decimal fraction for the following. Indicate the number of tenths and hundredths.

a. $6\frac{52}{100}$ = ________ d. $1\frac{9}{100}$ = ________

b. $3\frac{3}{10}$ = ________ e. $\frac{9}{10}$ = ________

c. $\frac{90}{100}$ = ________ f. $\frac{1}{2}$ = ________

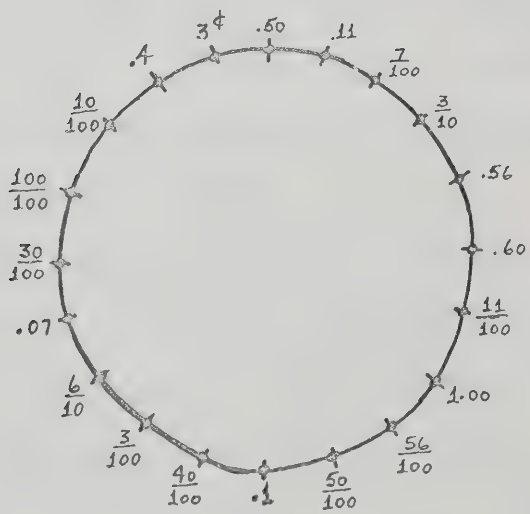
6. Change the decimal fractions to fractions with denominators 100.

a. 2.4 = _____ d. 5.75 = ________

b. 6.66 = _____ e. 10.30 = ________

c. 0.07 = _____ f. 3.01 = ________

7.



Around the circle you will see the same numeral written in different ways. With a ruler draw lines to join points with the same number.

IV. Bob read that Edmonton had 2.34 inches of snowfall in December and 3.78 inches of snowfall in January. Edmonton had a total of how many inches of snow in these two months?

$2.34 + 3.78 = N$ You must find the numeral to replace N.

(a)
```
   1
   2.34
 + 3.78
 ------
      2
```
First add the hundredths' column. The sum is 12 hundredths. Think of 12 hundredths as 1 tenth and 2 hundredths. Write the 2 hundredths in the hundredths' place. Remember the 1 tenth.

(b)
```
   1 1
   2.34
 + 3.78
 ------
    .12
```
Add the tenths' column. The sum of 11 tenths. Think of 11 tenths as 1 one and 1 tenth. Write the 1 tenth in the tenths' place. Remember the 1 one.

(c)
```
   1 1
   2.34
 + 3.78
 ------
   6.12
```
Write the decimal point to separate the whole numbers and the fractional numbers. Add the ones' column. Record and read the sum.

Write an equation for the following problems and solve the equations. Use decimal fractions.

1. Barry earned $48.35 during the summer holidays. His friend Glen earned $84.75. How much did the two boys earn together?

2. George weighs 67.86 pounds and Bill weighs 25.32 pounds. How much do the two boys weigh?

3. The rainfall recorded for Edmonton during the months of June, July, and August was 1.76 in., 2.32 in., and 1.09 in. respectively. How many inches of rain fell in Edmonton during these three months?

4. Make up some questions of your own and let a friend try to solve them.

APPENDIX E

PRE-TEST

PRE-TEST

Schools: ______________________________

Name: ______________________________

Directions:

Read each question carefully and decide which of the four answers is the correct answer. Write the letter for this answer on the line beside the question.

Sample Items:

I. Which of the following numbers has the largest value?

a) 23 b) 9 c) 35 d) 11 I. ________

II. Which sign means add?

a) −

b) +

c) x

d) ÷ II. ________

1). Which numeral has a 7 in the hundred place:

a) 3571 b) 7253 c) 2357 d) 2753 1.________

2). For 2.7 + 1.35 = N. the value for N is:

a) 4.05 b) 3.05 c) 2.05 d) 1.62 2.________

3). In the numeral 47,721 the value of the 7 in the thousand place is:

a) 10 times the value of the 7 in the hundred place

b) 100 times the value of the 7 in the hundred place

c) $\frac{1}{10}$ of the value of the 7 in the hundred place.

d) the same as the value of the 7 in the hundred place 3. ________

4). Another way to express 0.62 is:

a) 0.6 + 0.2

b) 0.6 + 0.02

c) 0.6 + 0.002

d) 0.6 + 0.1 + 0.1 4. ________

5). How would you write the decimal ninety and nine-hundredths?

a) .9009 b) 90.900 c) 90.09 d) 90.009 5. ________

6). Another way to express 4 x (60 + 7) is:

a) 4 + (60 x 7)

b) (4 + 7) x 60

c) (4 + 60) x (4 + 7)

d) (4 x 60) + (4 x 7) 6. ________

7).
$$\begin{array}{r} 347 \\ \times\ 2 \\ \hline 14 \\ 80 \\ 600 \\ \hline 694 \end{array}$$

The 80 is the product of:

a) 2 x 7

b) 4 x 20

c) 8 x 10

d) 2 x 40 7. ________

8). Which would give the correct answer to 563 x 39?

a) Multiply 563 x 3; 563 x 9; then add the answers.

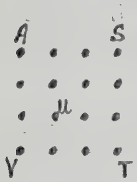
b) Multiply 563 x 3; 563 x 39; then add the answers.

c) Multiply 39 x 3; 39 x 60; 39 x 500; then add the answers.

d) Multiply 39 x 3; 39 x 63; 39 x 563; then add the answers.

8. ____________

9).

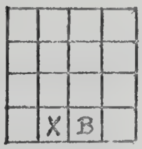


If (1,1) is the number pair name for the point A, then (4,4) should be the name for point:

a) S b) T c) U d) V

9. ____________

10).

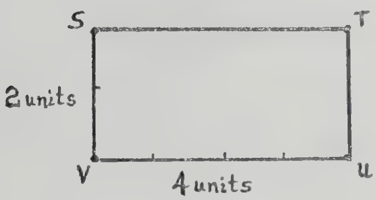


If the number pair (2,1) is assigned to the square labelled B, what number pair should be assigned to the square labelled X?

a) (1,3) b) (3,1) c) (1,2) d) (2,1)

10. ____________

11). What is the area of the region STUV?



a) 16 square units

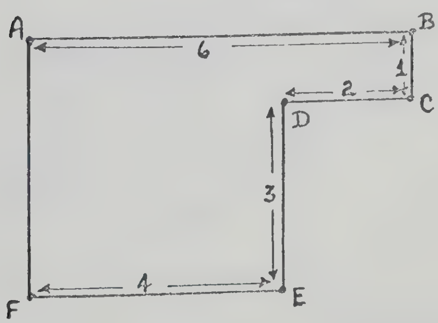
b) 12 square units

c) 8 square units

d) 6 square units

11. ____________

12). The area of the region ABCDEF is:



a) 12 square units

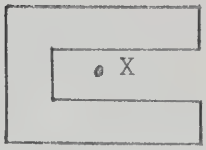
b) 16 square units

c) 18 square units

d) 24 square units

12. ____________

13). The point X is:



a) inside a closed curve

b) inside a rectangle

c) outside a rectangle

d) outside a closed curve 13. ________

14). Which set has only even members:

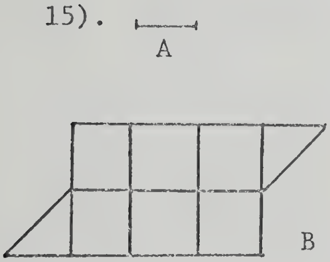
a) (2, 4, 6, 10)

b) (8, 12, 13, 15)

c) (3, 5, 7, 9)

d) (5, 7, 14, 18) 14. ________

15).



If the unit of measure is A, how many units could fit around figure B?

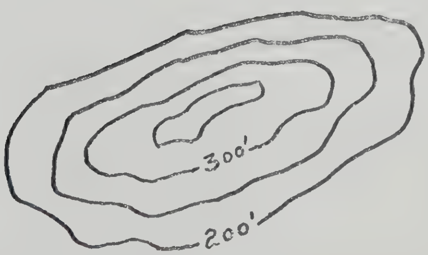
a) 7 units

b) 8 units

c) 10 units

d) more than 12 units 15. ________

16).



The height of the hill shown is:

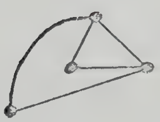
a) about 250'

b) 300'

c) about 350'

d) 400' 16. ________

17).



The network shown divides the plane represented by this page into several regions. How many regions are there?

a) 2 b) 3 c) 4 d) 1 17. ________

R.F. = 1:50,000

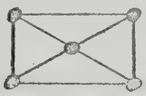
18). The scale on a map is 1:50,000. That means that:

a) 1 mile on the map is 50,000 miles on the earth.

b) 1 foot on the map is 50,000 feet on the earth.

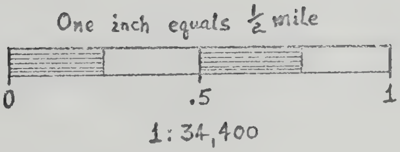
c) 1 inch on the map is 50,000 inches on the earth.

d) 1 inch on the map is 50,000 feet on the earch. 18. ________

19). 

How many segments are there in the network shown?

a) 5 b) 6 c) 7 d) 8 19. ________

20). 

This scale is from a map. On this map two towns are 2 inches apart which is equal to:

a) .5 miles b) 1 mile c) 2 miles d) 31,400 miles

20. ________